

Finite Dimensional Internal Model-Based Repetitive Control of Nonlinear Passive Systems

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Abstract—In this paper a new class of globally stable finite dimensional repetitive controllers for nonlinear passive systems is proposed. The passivity-based design of the proposed repetitive controller avoids the problem of tight stability conditions and slow convergence of the conventional, internal model-based, repetitive controllers. The existing internal and external model-based repetitive controllers can be derived as the special cases of the proposed controller. Also, it is possible a simple extension of the basic algorithm by an adaptive notch filter which provide asymptotic tracking of periodic signal with unknown signal frequency. The simulation results on a two degrees of freedom planar elbow manipulator illustrate the controller performances.

I. INTRODUCTION

Many tracking systems, such as computer disk drives [1], rotation machine tools [2], or robots [3], have to deal with periodic reference and/or disturbance signals. A promising control approach to achieving the tracking of periodic reference signals is learning control or repetitive control. In contrast with the conventional approaches to trajectory tracking control, repetitive control schemes are easy to implement and do not require the exact knowledge of the dynamic model.

Repetitive controllers can be classified as being either internal model-based or external model-based [4]. Controllers using the internal model are linear and have periodic signal generators [5], [6]. In the external model controllers the disturbance model is placed outside the basic feedback loop [3], [7].

The internal model controllers are based on a delayed integral action which produces an infinite number of poles on imaginary axes. However, the asymptotic convergence can only be guaranteed under restrictive conditions in the plant dynamics. Further, the positive feedback loop used to generate the periodic signal decreases the stability margin. So, the repetitive controller is likely to make the system unstable. To enhance the robustness of these repetitive control schemes, the repetitive update rule is modified to include the so-called Q-filter [5], [6]. Unfortunately, the use of the Q-filter eliminates the ability of tracking errors to converge to zero. Therefore, the trade-off between stability and tracking performance has been considered to be an important factor in the repetitive control system.

Another problem is that, due to infinite dimensional dynamics of delayed line, a large memory space is required

for digital implementation of the control law. To overcome this problem, in [8] a finite dimensional approximation of delayed line is proposed in the form of a cascade connection of N harmonic oscillators and one integrator.

The advantages of the internal model controllers are that they are linear, making analysis and implementation easier. The disadvantages are that the stability is almost entirely governed by the feedback loop of the repetitive compensator. The frequency response of the system is altered and robustness to noise and unmodelled dynamics is reduced.

The external model controllers are based on the feedforward compensation of inverse dynamics. The disturbance model is adjusted adaptively to match the actual disturbance. The central idea in [3] is that the disturbance can be represented as a linear combination of basis functions like Fourier series expansion. In this way, an adaptive control law with regressor matrix containing basis functions is obtained. In [7] unknown disturbance functions are represented by integral equations of the first kind involving a known kernel and unknown influence functions. The learning rule indirectly estimates the unknown disturbance function by updating the influence function.

The main advantage of the external model approach is that there is no significant influence on the stability conditions of the control system. The map between the feedforward function error and the tracking errors is strictly passive. Thus, the control system is robust to the imprecise estimation of the robot inverse dynamics. The disadvantage is that the analysis and implementation are more complex than for the internal model-based algorithms.

In this paper a new class of internal model-based repetitive controllers for nonlinear passive systems is proposed. The proposed finite dimensional repetitive controller is founded on the passivity-based design and has a structure in the form of a parallel connection of N linear oscillators and one integrator. The passive interconnection of the proposed controller with nonlinear passive systems has the same stability conditions as the controller with the exact feedforward compensation of nonlinear systems dynamics.

This paper is organized as follows. The problem formulation and stability analysis are presented in Section II and III. The passivity properties of the proposed controllers are considered in Section IV. The equivalence between internal

and external repetitive controllers is shown in Section V. The extension of the proposed controller with the frequency estimator is presented in Section VI. The simulation results are presented in Section VII. Finally, the concluding remarks are emphasized in Section VIII.

II. REPETITIVE LEARNING CONTROL

A. Problem Formulation

Consider the error dynamics of a learning control task described by the following state equation examined in [7] and [9]

$$\dot{e} = f(e, t) + B(e, t)[w(t) - \hat{w}(t)], \quad (1)$$

where $e(t) \in \mathbb{R}^n$ is the error between state and reference/disturbance vector, $w(t) \in \mathbb{R}^m$ is an unknown nonlinear function (desired inverse dynamics) and $\hat{w}(t) \in \mathbb{R}^m$ is an estimate of $w(t)$. In other words, $\hat{w}(t)$ provides feed-forward compensation of the desired inverse dynamics $w(t)$. The functions $f(e, t) \in \mathbb{R}^n$ and $B(e, t) \in \mathbb{R}^{n \times m}$ are bounded when $e(t)$ is bounded.

In a similar manner as [7], we suppose that (1) satisfies the following assumptions.

Assumption 1: The origin of the error system $e(t) = 0$ is uniformly asymptotically stable for

$$\dot{e} = f(e, t). \quad (2)$$

Moreover, there exist a first-order differentiable, positive-definite function $V_1(e, t) \in \mathbb{R}$, a positive-definite matrix $Q \in \mathbb{R}^{n \times n}$, and a known matrix $R \in \mathbb{R}^{n \times m}$ such that

$$\dot{V}_1(e, t) \leq -e^T Q e + e^T R \tilde{w}(t), \quad (3)$$

where $\tilde{w}(t) \in \mathbb{R}^m$ is the estimation error term defined by

$$\tilde{w}(t) = w(t) - \hat{w}(t). \quad (4)$$

Assumption 2: The unknown nonlinear function $w(t)$ is periodic with a known period T , so that $w(t) = w(t - T)$. Further, the function $w(t)$ can be represented by the infinite Fourier series expansion

$$w(t) = \sum_{k=0}^{\infty} [\bar{a}_k \cos(k\omega t) + \bar{b}_k \sin(k\omega t)], \quad (5)$$

where $\bar{a}_k, \bar{b}_k \in \mathbb{R}^m$ are unknown constant vectors.

Assumption 3: The periodic reference trajectory $x_d(t)$ with the period T can be represented in the form of Fourier series

$$x_d(t) = \sum_{k=0}^{\bar{N}} [a_k \cos(k\omega t) + b_k \sin(k\omega t)], \quad (6)$$

where $\omega = \frac{2\pi}{T}$ is the fundamental frequency, and $a_0, a_k, b_k \in \mathbb{R}^n$ are constant vectors.

The control objective for the general problem given in (1) is to design a finite dimensional learning-based estimate $\hat{w}(t)$ such that error $e(t)$ for $t \rightarrow \infty$ can be made arbitrary small.

B. Class of Finite-Dimensional Repetitive Controllers

We consider the control law given by

$$\hat{w}(t) = k_c R^T e + Q_0 z_0 + \sum_{k=1}^N Q_k \dot{z}_k, \quad (7)$$

$$\ddot{z}_k + k^2 \omega^2 z_k = Q_k R^T e, \quad k = 1, \dots, N, \quad (8)$$

$$\dot{z}_0 = Q_0 R^T e, \quad (9)$$

where $Q_k \in \mathbb{R}^{m \times m}$ ($k = 0, \dots, N$) are constant positive-definite diagonal matrix and $k_c \in \mathbb{R}$ is a positive constant control gain.

The parallel interconnection of the N harmonic oscillators (8) and the integrator (9) represents the internal model of the periodic reference signal $x_d(t)$ including higher order harmonics which are induced by the nonlinear system dynamics, so that the condition $N \geq \bar{N}$ must be satisfied.

III. STABILITY ANALYSIS

A. Error Equations

Introducing the change of variables $\tilde{z}_k = z_k - z_k^*$, $k = 0, 1, \dots, N$, with

$$z_0^* = Q_0^{-1} \bar{a}_0, \quad (10)$$

$$z_k^* = k^{-1} \omega^{-1} Q_k^{-1} [\bar{a}_k \sin(k\omega t) - \bar{b}_k \cos(k\omega t)], \quad (11)$$

the following error equations are obtained

$$\dot{e} = f(e, t) - B(e, t) \bar{w} + B(e, t) d(t), \quad (12)$$

$$\bar{w} = k_c R^T e + Q_0 \tilde{z}_0 + \sum_{k=1}^N Q_k \dot{\tilde{z}}_k, \quad (13)$$

$$\ddot{\tilde{z}}_k + k^2 \omega^2 \tilde{z}_k = Q_k R^T e, \quad k = 1, \dots, N, \quad (14)$$

$$\dot{\tilde{z}}_0 = Q_0 R^T e, \quad (15)$$

where we used the Fourier series expansion (5) of the function $w(t)$ and property $\ddot{z}_k^* + k^2 \omega^2 z_k^* = 0$. The function $d(t)$ in (12) is the error in estimation of the desired robot inverse dynamics which consists harmonics of $N + 1$ order

$$d(t) = \sum_{k=N+1}^{\infty} [\bar{a}_k \cos(k\omega t) + \bar{b}_k \sin(k\omega t)]. \quad (16)$$

From the equation (16) we can conclude that the tracking error has zero harmonic content at the repetitive frequency and its harmonics up to N (where N is the number of harmonic oscillators in the controller). Also, the bound on the tracking error decreases with N . In the limit $N \rightarrow \infty$ the above model of the repetitive controller works as well as the ideal infinity dimensional model in achieving perfect tracking of periodic reference signals [10]. Note that this conclusion is valid only for continuously differentiable periodic reference signals.

B. Lyapunov-Based Stability Analysis

We consider stability of the unperturbed systems (12)-(15), where $d(t) = 0$, by the Lyapunov's direct method. First, we propose appropriate Lyapunov function candidate. Then, global stability conditions on the controller gains are established. Finally, LaSalle invariance principle is invoked to guarantee the asymptotic stability.

1) *Lyapunov function candidate*: We define the Lyapunov function candidate $V = V_1 + V_2$, where $V_1(e, t)$ satisfies assumption (3) and V_2 is defined by

$$V_2 = \frac{1}{2} \sum_{k=1}^N \dot{z}_k^T \dot{z}_k + \frac{1}{2} \omega^2 \sum_{k=1}^N k^2 \ddot{z}_k^T \ddot{z}_k + \frac{1}{2} \dot{z}_0^T \dot{z}_0. \quad (17)$$

Taking the derivative of V with respect to time yields

$$\begin{aligned} \dot{V} \leq & -e^T Q e - e^T R \bar{w}(t) + \sum_{k=1}^N \dot{z}_k^T \ddot{z}_k + \omega^2 \sum_{k=1}^N k^2 \ddot{z}_k^T \dot{z}_k + \\ & + \dot{z}_0^T \ddot{z}_0 \end{aligned} \quad (18)$$

Substituting (13)-(15) in the previous equation, we obtain

$$\begin{aligned} \dot{V} \leq & -e^T Q e - e^T R \left(k_c R^T e + Q_0 \dot{z}_0 + \sum_{k=1}^N Q_k \dot{z}_k \right) + \\ & + \sum_{k=1}^N \dot{z}_k^T (-k^2 \omega^2 \ddot{z}_k + Q_k R^T e) + \omega^2 \sum_{k=1}^N k^2 \ddot{z}_k^T \dot{z}_k + \\ & + \dot{z}_0^T Q_0 R^T e = -e^T (Q + k_c R R^T) e \leq -e^T Q e \end{aligned} \quad (19)$$

2) *LaSalle invariance principle*: Since the time derivative of Lyapunov function \dot{V} is not a negative definite function then only a negative semi-definite one, we must apply LaSalle invariance principle to conclude asymptotic stability. It remains to prove that the maximal invariant set of (12)-(15) contained in the set

$$\Omega = \{x \in \mathbb{R}^{n+(2N+1)m} \mid \dot{V}(x) = 0\}, \quad (20)$$

consists of the origin $x = [e^T \ \dot{z}_0^T \ \dot{z}_1^T \ \ddot{z}_1^T \ \dots \ \dot{z}_N^T \ \ddot{z}_N^T]^T = 0$. Since $\dot{V}(x) = 0$ means $e = 0$, substitution of $\dot{e} = 0$ and $e = 0$ into (12)-(15) leads to

$$Q_0 \dot{z}_0 + \sum_{k=1}^N Q_k \dot{z}_k = 0, \quad (21)$$

$$\dot{z}_0 = 0, \quad \ddot{z}_k + k^2 \omega^2 \ddot{z}_k = 0, \quad k = 1, \dots, N. \quad (22)$$

The following step is to show that the differential-algebraic system (21)-(22) has only trivial solution $\dot{z}_0 = 0$, $\dot{z}_k = \ddot{z}_k = 0$, where $k = 1, \dots, N$. We prove that by using contradiction. Suppose that there exists a solution of differential equations (22), $\dot{z}_0 = C_1$ and $\dot{z}_k = C_2 \sin(k\omega t) + C_3 \cos(k\omega t)$, $k = 1, \dots, N$, where C_1 , C_2 and C_3 are some constant vectors. Inserting this solutions in equality (21) we get

$$Q_0 C_1 + \omega \sum_{k=1}^N k Q_k [C_2 \cos(k\omega t) - C_3 \sin(k\omega t)] = 0. \quad (23)$$

In the above mentioned expression, the right side of equality is a sum of linearly independent functions, which can not be equal to zero except for $C_1 = C_2 = C_3 = 0$, in other words, for $\dot{z}_0 = \dot{z}_k = \ddot{z}_k = 0$, where $k = 1, \dots, N$.

So, since the maximal invariant set in $\mathbb{R}^{n+(2N+1)m}$ is composed only of the origin, we conclude that the origin is asymptotically stable.

IV. PASSIVITY PROPERTIES OF REPETITIVE CONTROLLER

Unknown and unmodeled nonlinearities play an important role in the high-precision control. Any cancellation of nonlinearities by feedback which is not exact, may produce undesirable closed-loop behavior like large tracking errors, limit cycles and stick-slip motion. In contrast with model-dependent controllers, the passivity-based controllers are robust to model uncertainties and external disturbances. In this section we prove the passivity properties of the proposed repetitive controller.

Proposition 1. The error dynamics

$$\dot{e} = f(e, t) - B(e, t)u_1 + B(e, t)w_1, \quad (24)$$

in closed-loop with

$$u_1 = k_c R^T e, \quad (25)$$

is output strictly passive from the input torque w_1 to the output $y_1 = R^T e$, with a radially unbounded positive definite storage function V_1 defined by (3), i.e.

$$w_1^T y_1 \geq \dot{V}_1(e, t) + \delta \|y_1\|^2, \quad (26)$$

where $\delta = k_c$.

Proof. Taking the derivative of V_1 with respect to time yields

$$\begin{aligned} \dot{V}_1 \leq & -e^T Q e + e^T R(-u_1 + w_1) = \\ & = -e^T Q e - k_c e^T R R^T e + e^T R w_1 = \\ & = -e^T Q e - k_c y_1^T y_1 + y_1^T w_1, \end{aligned} \quad (27)$$

where we used (24) and definition of the output y_1 . From the last line of expression (27) directly follows (26).

Proposition 2. The system

$$\ddot{z}_k + k^2 \omega^2 \ddot{z}_k = Q_k w_2, \quad k = 1, \dots, N, \quad (28)$$

$$\dot{z}_0 = Q_0 w_2, \quad (29)$$

is passive from the input w_2 to the output $y_2 = Q_0 z_0 + \sum_{k=1}^N Q_k \dot{z}_k$ with a radially unbounded positive definite storage function V_2 defined by (17),

$$w_2^T y_2 \geq \dot{V}_2(\dot{z}_0, \dot{z}_1, \dot{z}_1, \dots, \dot{z}_N, \dot{z}_N). \quad (30)$$

Proof. Taking the derivative of V_1 with respect to time yields

$$\begin{aligned} \dot{V}_2 = & \sum_{k=1}^N \dot{z}_k^T \ddot{z}_k + \omega^2 \sum_{k=1}^N k^2 \ddot{z}_k^T \dot{z}_k + \dot{z}_0^T \ddot{z}_0 = \\ & = \sum_{k=1}^N \dot{z}_k^T (-k^2 \omega^2 \ddot{z}_k + Q_k w_2) + \omega^2 \sum_{k=1}^N k^2 \ddot{z}_k^T \dot{z}_k + \\ & + \dot{z}_0^T Q_0 w_2 = w_2^T \left(Q_0 z_0 + \sum_{k=1}^N Q_k \dot{z}_k \right) = w_2^T y_2, \end{aligned} \quad (31)$$

where we used (28), (29) and definition of the output y_2 . From the last line of expression (31) directly follows (30).

Proposition 3. The feedback interconnection of the system (24)-(25) with the system (28)-(29),

$$w_1 = -y_2 + w, \quad w_2 = y_1, \quad (32)$$

is output strictly passive from the input torque w to the output $y_1 = R^T e$, with a radially unbounded positive definite storage function $V = V_1 + V_2$,

$$w^T y_1 \geq \dot{V} + \delta \|y_1\|^2, \quad (33)$$

where where $\delta = k_c$.

Proof. Inserting $w_1 = -y_2 + w$, $w_2 = y_1$ in (26) and (30) we get (33).

Proposition 4. The feedback interconnection of the system (24)-(25) with the system (28)-(29) has finite \mathcal{L}_2 gain $\gamma \leq \frac{1}{\delta}$ where $\delta = k_c$. For proof see e.g. [11].

From the above mentioned propositions follows two important properties of the proposed repetitive controller. First, comparing expression (33) with (26) and (30) we can conclude that the repetitive controller preserve the closed loop stability. In other words, the passive feedback interconnection between the proposed repetitive controller and robot dynamics doesn't decrease stability margin which is characteristics for classical internal model based repetitive controllers [5] and their finite dimensional representations [8]. Second, the map between the inverse dynamics estimation error $d(t)$ and output error $R^T e$ is strictly passive which means that the control system is robust to the imprecise estimation of the robot inverse dynamics.

V. EQUIVALENCE BETWEEN INTERNAL AND EXTERNAL REPETITIVE CONTROLLERS

In this section we show that the proposed control law (7)-(9) provides a unified framework for the existing internal and external repetitive controllers. Although the internal and external repetitive controllers are based on different control strategies and their control algorithms seem very different, they can be considered as the special cases of the repetitive controller (7)-(9).

A. Kernel-Based Representation

The equation (8) can be rewritten as

$$\begin{bmatrix} \dot{x}_{1k} \\ \dot{x}_{2k} \end{bmatrix} = \begin{bmatrix} O & I \\ -k^2 \omega^2 I & O \end{bmatrix} \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix} + \begin{bmatrix} O \\ Q_k \end{bmatrix} y, \quad (34)$$

where $x_{1k} = z_k$, $x_{2k} = \dot{z}_k$, $y = R^T e$, $I \in \mathbb{R}^{m \times m}$ is the unit matrix and $O \in \mathbb{R}^{m \times m}$ is the zero matrix.

The solution of (34) for the initial conditions $x_{1k} = x_{2k} = 0$ is

$$\begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix} = \int_0^t e^{A_k(t-\tau)} B_k y(\tau) d\tau, \quad (35)$$

where

$$A_k = \begin{bmatrix} O & I \\ -k^2 \omega^2 I & O \end{bmatrix}, \quad B_k = \begin{bmatrix} O \\ Q_k \end{bmatrix}. \quad (36)$$

The matrix exponential can be evaluated as

$$e^{A_k t} = \begin{bmatrix} \cos(k\omega t) I & \frac{1}{k\omega} \sin(k\omega t) I \\ -k\omega \sin(k\omega t) I & \cos(k\omega t) I \end{bmatrix}, \quad (37)$$

so that finally we get

$$x_{2k} = \int_0^t Q_k \cos[k\omega(t-\tau)] y(\tau) d\tau. \quad (38)$$

Inserting (38) in (7) we get

$$\hat{w}(t) = k_c y(t) + \sum_{k=0}^N Q_k^2 \int_0^t \cos[k\omega(t-\tau)] y(\tau) d\tau. \quad (39)$$

The above-mentioned expression can be rewritten as

$$\hat{w}(t) = k_c y(t) + \int_0^t K(t, \tau) y(\tau) d\tau. \quad (40)$$

where we used notation

$$K(t, \tau) = \sum_{k=0}^N Q_k^2 \cos[k\omega(t-\tau)]. \quad (41)$$

Note that the learning algorithm (40) with (41) can be considered as a simplified version of the kernel-based repetitive controller [7].

B. Infinity Dimensional Internal Repetitive Controllers

For the choice $Q_0^2 = \frac{k_c}{T} I$, $Q_k^2 = \frac{2k_c}{T} I$ where $k = 1, 2, \dots$ and $N \rightarrow \infty$ the function $K(t, \tau)$ becomes the Dirac comb function

$$K(t, \tau) = k_c I \sum_{k=-\infty}^{\infty} \delta(t - \tau - kT), \quad (42)$$

where we used the Fourier representation of the Dirac comb

$$\sum_{k=-\infty}^{\infty} \delta(t - \tau - kT) = \frac{1}{T} + \frac{2}{T} \sum_{k=1}^{\infty} \cos[k\omega(t - \tau)]. \quad (43)$$

Inserting (42) in (40) and using properties of the Dirac function, the following expression is obtained

$$\hat{w}(t) = k_c y(t) + k_c \sum_{k=1}^{\infty} y(t - kT), \quad (44)$$

or

$$\hat{w}(t) = \hat{w}(t - T) + k_c y(t). \quad (45)$$

The control law (45) is the standard repetitive update rule proposed by [5]. From the expression (44) follows that the standard repetitive update rule cannot guaranty boundedness of $\hat{w}(t)$. To address the boundedness problem associated with the standard repetitive update rule, saturated versions of the update rule (45) are proposed in [10] and [9].

C. External Repetitive Controllers

Using the addition formula for cosine function, the expression (39) can be rewritten as

$$\begin{aligned} \hat{w}(t) = & \sum_{k=0}^N Q_k^2 \left(\sin(k\omega t) \int_0^t \sin(k\omega\tau) y(\tau) d\tau + \right. \\ & \left. + \cos(k\omega t) \int_0^t \cos(k\omega\tau) y(\tau) d\tau \right) + k_c y(t). \end{aligned} \quad (46)$$

The above mentioned expression can be rewritten as

$$\hat{w}(t) = \sum_{k=0}^N \left(\theta_k^{(1)} \sin(k\omega t) + \theta_k^{(2)} \cos(k\omega t) \right) + k_c y(t), \quad (47)$$

where

$$\dot{\theta}_k^{(1)} = Q_k^2 \sin(k\omega t) y(t), \quad (48)$$

$$\dot{\theta}_k^{(2)} = Q_k^2 \cos(k\omega t) y(t), \quad (49)$$

for $k = 0, 1, \dots, N$. The adaptive control law (47)-(49) can be expressed in the more compact form

$$\hat{w}(t) = \sum_{k=0}^{2N} \theta_k \phi_k(t) + k_c y(t), \quad (50)$$

$$\dot{\theta}_k = Q_k^2 \phi_k(t) y(t), \quad (51)$$

for $k = 0, 1, \dots, 2N$, where

$$\phi_k(t) = \begin{cases} \cos[k\omega t] & \text{if } k \leq N \\ \sin[(k - N)\omega t] & \text{if } k > N \end{cases}. \quad (52)$$

The adaptive algorithm (50) and (51) with (52) is known as the Desired Compensation Learning Law [12], the Basis Function Algorithm [4] or the Repetitive Fourier Controller [13].

VI. REPETITIVE CONTROL WITH UNKNOWN FUNDAMENTAL FREQUENCY

The main drawbacks of repetitive control is the requirement of exact knowledge of the period-time of the external reference/disturbance signals. In literature several solutions have been proposed, most of them considering an adaptive scheme to estimate the period-time on the base of gradient minimization of a cost function [14] or using multiple memory-loops with aim to decrease the sensitivity to small changes in signal frequency [15]. Unfortunately, the mentioned algorithms are relatively complex for implementation and cannot guaranty perfect asymptotic tracking.

Another possible approach to repetitive control for tracking periodic signals with unknown frequency is by using frequency estimators. There exist several algorithms for online estimation of the frequency of a sinusoidal signal. The most of them are based on the Regalia notch filter [16]. The first continuous version of the adaptive notch filter is proposed in [17], and first globally convergent frequency estimator is proposed in [18]. The above-mentioned algorithms cannot be applied to standard repetitive controllers because they provide frequency estimation of a single sinusoidal signal.

In [19] and [20] are proposed algorithms which provide estimation of frequencies of a multi-sinusoidal signal. The frequencies estimator proposed in [19] is of $5N - 1$ order while estimator proposed in [20] is of $3N$ order, where N is the number of different frequencies. So, for example, the finite dimensional repetitive controllers [12] or [8] in combination with frequencies estimator [20] are of $2Nn + n + 3N$ order.

The repetitive controller (7)-(9) is especially suitable for direct implementation of a simple adaptive notch filter of the first order

$$\dot{\omega} = -\gamma z_{1j} y_j, \quad (53)$$

where γ is a positive constant, z_{1j} is the j -th component of the vector z_1 in (8), y_j is the j -th component of the output vector $y = R^T e$.

The repetitive controller (7)-(9) in combination with the frequency estimator (53) is of $2Nn + n + 1$ order. It can be a significant reduction of the controller order, especially in the case of high precision repetitive control tasks, where we have $N \gg n$. The stability analysis of the repetitive controller (7)-(9) in combination with the frequency estimator (53) is currently under research.

VII. SIMULATION EXAMPLE

We consider a 2-DOF manipulator with revolute joints in vertical plane, considered in [21]. The numerical values of robot parameters have been taken from [22].

The desired periodic reference trajectories are

$$q_{d1} = \frac{1}{2} + \sum_{k=1}^3 \frac{\sin(k\omega t)}{k+1}, \quad q_{d2} = 1 - 2 \sum_{k=1}^3 \frac{\cos(k\omega t)}{k^2+1}, \quad (54)$$

where $\omega = 2 \text{ rad/s}$.

We applied the control law (7)-(9) with (53) where the expression (7) is extended by an additional term $k_D \|e\| e$, which provide global asymptotic stability [23], [24], so that assumption (3) is satisfied. The error vector is $e = [\tilde{q} \ \dot{\tilde{q}}]^T$ where \tilde{q} and $\dot{\tilde{q}}$ are robot position and velocity errors. The matrix R has the following form $R = [I \ \alpha I]^T$ so that passive output variable is $y = \dot{\tilde{q}} + \alpha \tilde{q}$.

The controller gains are chosen in accordance with stability conditions derived in [24], where the convergence properties of the proposed controllers are demonstrated for the case of known reference signal frequency.

Extending the repetitive controller (7)-(9) with frequency estimator (53) it is possible asymptotic tracking of the reference periodic signal with uncertain period-time. The fundamental frequency of the reference signal (54) is $\omega_r = 2 \text{ rad/s}$, and its estimate $\omega(0) = 2.5 \text{ rad/s}$ is the initial conditions for (53). The number of oscillators is $N = 12$.

Fig. 1. shows a comparison of the positions of the robot manipulators and reference signals. In Fig. 2. we can see comparison of the tracking errors for the repetitive controller (7)-(9) with constant fundamental frequency $\omega = 2.5 \text{ rad/s}$ and the repetitive controller (7)-(9) with frequency estimator (53). In contrast with the repetitive controller (7)-(9), the repetitive controller (7)-(9) with frequency estimator (53) shows exponential convergence toward an arbitrary small tracking error which depends only on the number of oscillators N . In Fig. 3. we can see convergence of the frequency estimate ω toward the fundamental frequency of the reference signal.

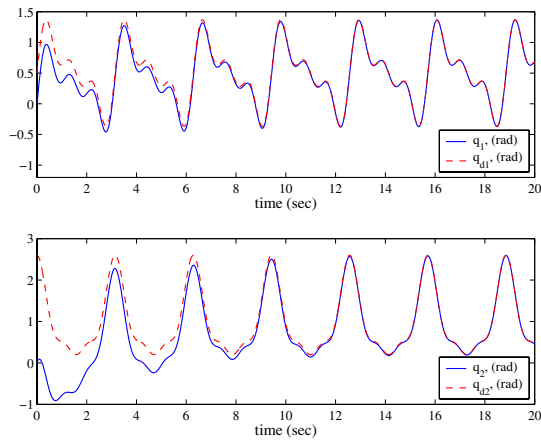


Fig. 1. The periodic reference signals and positions of robot manipulators.

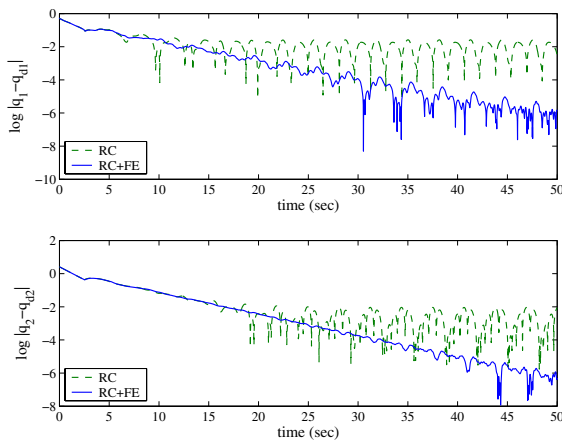


Fig. 2. The comparison of tracking errors for repetitive controller (RC) and repetitive controller with frequency estimator (RC+FE).

VIII. CONCLUSIONS

In this paper a new class of finite dimensional repetitive controllers for nonlinear passive systems is proposed. The proposed repetitive controller connects the main advantage of internal model controllers - implementation simplicity, with robustness based on passivity of external model controllers. In the case of periodic signals with unknown frequency, the proposed controller can be easily combined with the frequency estimators.

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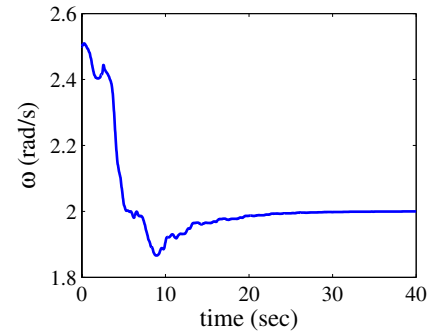


Fig. 3. The frequency estimate using the adaptive notch filter.

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