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Verification of Crankshaft Design and Fatigue Strength for in-line and v-type Marine Diesel Engines

***Nenad VULIĆ*[1], *Ante ŠESTAN*[2] *and Vedrana CVITANIĆ*[3]**

1) Hrvatski registar brodova (Croatian Register of Shipping), Marasovićeva 67 HR-21000 Split, **Croatia**
2) Fakultet strojarstva i brodogradnje Sveučilišta u Zagrebu (Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb), Ivana Lučića 5 HR-10000 Zagreb, **Croatia**
3) Fakultet elektrotehnike, strojarstva i brodogradnje Sveučilišta u Splitu (Faculty of Electrical Engineering and Naval Architecture), Ruđera Boškovića bb HR-21000 Split, **Croatia**

Nenad.Vulic@fesb.hr

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Design approval of marine Diesel engines crankshafts is, in general, based upon the Technical Rules of classification societies. These Rules regularly contain provisions of the IACS UR M53 document: Calculation of Crankshafts for Internal Combustion Engines. The acceptance criteria are related to fatigue strength. The main drawback of this document is that an essential part of the procedure is not defined – how to determine kinematic values and which forces shall be taken into account in the simplified single crank reciprocating mechanism model of in-line and V-type engines. Additionally, as the design approval starts from the cylinder pressure vs. crank angle curve submitted by the engine manufacturer, these values can be used for direct evaluation of torsional vibration excitation forces.

This paper aims to fill the above-mentioned gap, presenting the procedure systematised by the authors to evaluate all the kinematic values and forces relevant for crankshaft fatigue strength validation. The basic idea is to start the analysis from the connecting rod, instead of the piston. The trigonometric approximation procedure for cylinder pressures (necessary for V-type engines crankshaft fatigue strength) and for tangential forces (necessary to determine torsional vibration excitation) is also presented. The entire calculation is illustrated on a selected actual engine example.

Provjera konstrukcijskog oblika i zamorne čvrstoće koljenastog vratila rednih i V brodskih dizel motora

Izvornoznanstveni članak

Klasifikacijska društva u pravilu osnivaju odobrenje konstrukcije koljenastog vratila na svojim Tehničkim pravilima. Pravila uobičajeno sadrže odredbe dokumenta IACS UR M53: Proračun koljenastog vratila brodskih dizelskih motora. Kriteriji prihvatljivosti su povezani sa zamornom čvrstoćom. Glavni nedostatak ovog dokumenta je to što bitan dio cjelovitog postupka njime nije određen – kako odrediti kinematičke veličine, te koje sile treba uzeti u obzir u pojednostavljenom modelu stapnog mehanizma s jednim koljenom za redne i V motore. Dodatno k tome, budući da odobrenje započinje od krivulje tlakova u cilindru o ovisnosti o kutu osnog koljena, koju dostavlja proizvođač motora, ove se veličine mogu koristiti u neposrednom proračunu uzbudnih sila torzijskih vibracija.

Ovim se radom teži popuniti navedenu prazninu, prikazom postupka, kojega su autori uobličili, za određivanje svih kinematičkih veličina i sila značajnih za provjeru zamorne čvrstoće koljenastog vratila. Temeljna je zamisao započeti od analize ojnice, umjesto klipa ili stapa. Prikazan je i postupak trigonometrijske aproksimacije tlakova u cilindru (neophodnih za zamornu čvrstoću koljenastog vratila V motora), kao i obodnih sila (neophodnih za izračun pobude torzijskih vibracija). Cjeloviti je proračun prikazan na izabranom primjeru stvarnoga motora.

1 Introduction

Each propulsion marine Diesel engine shall be tested and certified in order to be used onboard a ship. The necessary pre-condition for certification is the approval of the engine prototype. This process is commonly called the type approval and performed under supervision of classification societies. The engine type approval procedure consists of several phases: approval of documentation and the type test program, supervision during the type test of the engine, review of the obtained results and issuance of the Type Approval Certificate [1]. Classification society in charge performs all these steps. The essential step in the documentation approval phase is the approval of crankshaft design, with respect to its fatigue strength. A failure in this phase may have an adverse effect in engine project, construction, production, testing and especially in its exploitation.

Understanding the importance of the crankshaft design approval, the International Association of Classification Societies (IACS), in close cooperation with the International Council on Combustion Engines (CIMAC, the engine builders' association), developed the Unified Requirement UR M53 – Calculation of Crankshafts for Internal Combustion Engines in 1986. This document was revised in 2004, to take into account some important

Symbols/Oznake

a_p – piston acceleration, m/s²
– ubrzanje klipa/stapa

A_k – cosine Fourier coefficient
– kosinusni Fourier-ov koeficijent

B – crankweb width, mm
– širina ramena osnoga koljena

B_k – sine Fourier coefficient
– sinusni Fourier-ov koeficijen

d – engine cylinder bore, m
– promjer cilindra motora

d_x – crank pin diameter (*D*), or crank journal diameter (D_G), mm
– promjer osnaca ležaja ojnice (*D*), ili promjer temeljnog osnaca (D_G),

F – active or inertial force, N
– aktivna ili inercijska sila

F_r – resultant force acting on the crank web perpendicular to its axis of rotation in radial direction, N
– rezultantna sila na osno koljeno okomito na njegovu os vrtnje u radijalnom smjeru

F_t – resultant force acting on the crank web perpendicular to its axis of rotation in tangential direction, N
– rezultantna sila na osno koljeno okomito na njegovu os vrtnje u obodnom smjeru

g – acceleration of gravity, m/s²
– ubrzanje sile teže

$G=mg$ – weight, N
– težina

i_{CR} – radius of inertia of connecting rod about its centre of gravity (COG), m
– polumjer tromosti mase ojnice oko težišta

J_{CR} – mass moment of inertia of the connecting rod around its centre of gravity (COG), kgm²
– moment tromosti mase ojnice oko težišta

K – factor of crankshaft production process
– faktor postupka proizvodnje koljenastog vratila

K_e – factor of influence of adjacent crank and bearing restraint
– faktor utjecaja susjednih koljena i rubnih uvjeta u ležajevima

ℓ – length of connecting rod, m
– duljina ojnice

ℓ_R – distance of connecting rod centre of gravity to the crank pin centre, m
– udaljenost težišta ojnice od središta osnaca ležaja ojnice

L_1 – distance between main journal centre line and crankweb centre, m
– udaljenost između središnjih presjeka temeljnog ležaja i ramena osnog koljena

L_2 – distance between main journal centre line and connecting rod centre, m
– udaljenost između središnjih presjeka temeljnog ležaja i ojnice

L_3 – distance between two adjacent main journal centre lines, m
– udaljenost između središnjih presjeka dvaju susjednih temeljnih ležajeva

m – number of cylinder pressure function $p(\alpha)$ discrete points
– broj diskretnih točaka funkcije tlaka u cilindru $p(\alpha)$

m_{CR} – mass of connecting rod, kg
– masa ojnice

m_p – mass of reciprocating parts (piston, piston pin / piston rod and crosshead), kg
– masa dijelova u stapnom gibanju (klip i osovinica klipa, ili stap, stapna motka i križna glava)

m_W – mass of rotating parts (crank pin, crank web, counterweights and connecting bolts), kg
– masa rotirajućih dijelova (osnaca ležaja ojnice, osnoga koljena, temeljnog osnaca i protuutega)

M – active or inertial moment, Nm
– aktivni ili inercijski moment

M_{BrN} – bending moment, Nm
– moment savijanja

M_t – active moment acting on the crank around its axis of rotation, Nm
– aktivni moment na koljeno oko osi vrtnje

n – engine nominal speed, rpm
– nazivna brzina vrtnje motora

N – order of trigonometric approximation
– red trigonometrijske aproksimacije

p – cylinder pressure, N/m²
– tlak u cilindru

$p_{m,e}$ – mean effective pressure, N/m²
– prosječni efektivni tlak

$p_{m,i}$ – mean indicated pressure, N/m²
– prosječni indicirani tlak

$P_{m,i}$ – mean indicated power per cylinder, kW
– prosječna indicirana snaga po cilindru,

$P_{m,e}$ – mean effective power per cylinder, kW
– prosječna efektivna snaga po cilindru

Q_{rN} – shear force, N
– poprečna sila

r – crank radius (half of stroke), m
– polumjer osnoga koljena (polovina stapaja)

r_W – radial position of rotating parts common centre of gravity, m
– radijalni položaj zajedničkog težišta rotirajućih dijelova

r_X – fillet radius of crankpin (R_H), or fillet radius of crank journal (R_G), mm
– polumjer zaobljenja osnaca ležaja ojnice (R_H), ili polumjer zaobljenja temeljnog osnaca (R_G)

R – reactive force, N
– reaktivna sila

s_p – piston displacement measured from the top dead centre (TDC), m
– pomak klipa/stapa mjeren od gornje mrtve točke

$s=2\cdot r$ – stroke, m
– stapaj

t – time, s
– vrijeme

$v_m = s\cdot n/30$ – mean piston speed, m/s
– prosječna brzina klipa/stapa,

v_p – piston speed, m/s
– brzina klipa/stapa

W – crankweb thickness, mm
– debljina ramena osnog koljena

z – engine working cycle (z=2 for two-stroke; z=4 for four-stroke engines)
– radni ciklus motora (z=2 za dvotaktne; z=4 za četverotaktne motore)

α – crank angle (angular displacement), measured from the top dead centre (TDC), °
– kut osnoga koljena (kutni zakret), mjeren od gornje mrtve točke

α_B, α_T – crankpin fillet stress concentration factors for bending and torsion
– faktori koncentracije naprezanja pri savijanju i uvijanju za zaobljenje osnaca ležaja ojnice

β – connecting rod angular displacement, measured from the cylinder axis, °
– kutni zakret ojnice, mjeren od osi cilindra

β_B, β_Q, β_T – crank journal fillet stress concentration factors for bending, compression and torsion
– faktori koncentracije naprezanja pri savijanju, tlaku i uvijanju za zaobljenje temeljnoga osnaca

δ_m – mean absolute error of approximation
– prosječna apsolutna pogreška aproksimacije

δ_{max} – maximal absolute error of approximation
– najveća apsolutna pogreška aproksimacije

δ_s – mean square error of approximation
– srednja kvadratna pogreška aproksimacije

ε_{CR} – connecting rod angular acceleration, rad/s²
– kutno ubrzanje ojnice

η_{mech} – mechanical efficiency ratio,
– mehanički stupanj djelovanja

θ – angle of phase shift, defining the cylinders ignition sequence, °
– kut faznoga pomaka, koji određuje redoslijed paljenja u cilindrima

$\lambda=r/\ell$ – ratio of crank radius to the connecting rod length,
– omjer polumjera osnoga koljena i duljine ojnice

σ_{add} – additional bending stress due to misalignment, bedplate deformation, axial and bending vibrations, N/mm²
– dodatno savojno naprezanje uslijed pogrešaka centracije temeljne ploče i njenih deformacija, te uslijed uzdužnih i poprečnih vibracija

σ_B – minimal tensile strength of crankshaft material, N/mm²
– najmanja vlačna čvrstoća materijala koljenastoga vratila

σ_{BFN} – nominal alternating bending stress related to the crankweb, N/mm²
– nazivno naizmjenično naprezanje uslijed savijanja u ramenu osnoga koljena

σ_{DW} – allowable fatigue strength of crankshaft, N/mm²
– dozvoljena zamorna čvrstoća koljenastoga vratila

σ_{QFN} – nominal alternating compressive stress due to radial force related to the crankweb, N/mm²
– nazivno naizmjenično tlačno naprezanje u ramenu osnoga koljena uslijed radijalne sile

σ_V – equivalent alternating stress, N/mm²
– reducirano naizmjenično naprezanje,

τ_N – nominal alternating torsional stress referred to crankpin or crank journal, N/mm²
– nazivno naizmjenično naprezanje uslijed uvijanja u osnacu ležaja ojnice ili u temeljnome osnacu

φ_v – half of the angle between cylinder axes in case of V-engines
– polovina kuta između osi susjednih cilindara u V-motora

$\omega=\pi\cdot n/30$ – crankshaft speed of rotation, rad/s
– kutna brzina koljenastoga vratila,

ω_{CR} – connecting rod angular velocity, rad/s
– kutna brzina ojnice

outstanding issues in the original document. These are the influence of oil bores, the basis for evaluation of the stress concentration factors, as well details about the influence of production procedure and surface treatment [2].

Although the UR M53 (2004) aims to be a self-standing document, unfortunately this is not the case. Regardless of this, practically all of the internationally recognised classification societies base their Technical Rules on this document. The document takes into account all the necessary influences: design shape, dimensions, material and the operational loading of the single crank. However, the main outstanding issue in the UR M53 (2004), in the authors' opinion, is the fact that the operational loading calculation commences with the forces acting on the crankshaft. The document itself lacks a uniform and straightforward procedure how to determine these forces, from the data really provided by engine manufacturers: cylinder pressures vs. crank angle tables or curves, engine speed, as well as the dimensions and masses of the principal engine reciprocating mechanism components. If the document [2] had been extended by this procedure it would have been a much better basis for the approval of engine crankshaft design with respect to its fatigue strength, especially in the case of V-type engines.

The basic goal of this paper is to propose the necessary extension of the IACS UR M53 (2004) document. This extension shall provide a unified and straightforward procedure for calculation of the necessary kinematic values (displacements, velocities and accelerations), as well as the external active, external passive and inertial forces acting to the connecting rod, in order to determine loads onto a single crank. The task is to present the calculation of the kinematic and dynamic values in a general form: a unique procedure both for the in-line and for the V-type engines. Additionally, the paper presents the trigonometric approximation of cylinder pressures and the tangential forces, necessary for calculations of crankshaft strength in case of V-type engines and the calculation of torsional vibrations, the latter being beyond the scope of the paper.

Though the author(s) have already dealt with this subject in previously published papers [3, 4], the presented procedure had to be written from scratch. The present approach is much clearer and simpler than the previously published one. Textbooks [6-10], though very comprehensive, generally treat this problem implementing certain simplifications, which are avoided hereafter. Besides the basic approach to crankshaft approval, the paper presents the determination of kinematic values, dynamic values and briefly the fatigue strength calculations. The trigonometric approximation of pressures and tangential forces is given afterwards. Finally, a short description of the developed calculation program together with an actual engine example is also given. The authors' final aim is to make the Croatian Register of Shipping include the presented procedure into their Technical Rules, thus contributing to the general understanding of this approach and making the life of shafting designers and classification societies plan approval staff easier.

2 Crankshaft simplified model and the input data

In accordance with the IACS document UR M53 (2004) the simplified crankshaft model, consisting of a single crank, two neighbouring crank journals and the crank pin is sufficient for the calculation of the crankshaft fatigue strength. The influence of the adjacent cranks and bearing restraints to bending stresses is accounted for by the empirical factor K_e. However, the crankshaft constructive structural shape (with all the necessary details – the relevant dimensions), as well as the material properties is to be taken into account.

The basic input data are related to the engine cylinder / crank mechanism dimensions, masses and external loads. They are:

- working cycle (2–stroke, or 4–stroke),
- dimensions (d, r, ℓ, ℓ_R, r_W),
- masses and inertial properties (m_p, m_{CR}, i_{CR}, m_W), and
- loading related properties (g, $p = p(\alpha)$, n, η_{mech}, τ_N).

It shall be noted that the engine nominal speed n is assumed to be constant. Mass of reciprocating parts m_p (piston and piston pin – in case of a trunk piston engine, or piston, piston rod and crosshead – in case of a crosshead engine) is additional to (meaning: excludes) the mass of the connecting rod part in reciprocating motion. The mass of rotating parts m_W (crank pin, crank web and counterweights), together with r_W, does not affect forces acting on the connecting rod.

Working cycle, dimensions, masses and inertial properties are defined in the engine documentation submitted for approval. The engine manufacturer also submits the cylinder pressures vs. crank angles table. As the crank angles α are defined as measured from the top dead centre (TDC), the submitted $p(\alpha)$ data usually have to be modified to lie within the interval of $\alpha_{min/max} = \pm 180^{\circ}$ for two stroke engines, or $\pm 360^{\circ}$ for four stroke engines. The $p(\alpha)$ data shall be given in discrete form, for at least every 5° crank angle [1, 2].

The type approval of the engine does not prejudice or prescribe the configuration of the rest of the propulsion or auxiliary systems (gears, shafts, propellers or generator rotors). Consequently, the nominal alternating torsional stress τ_N shall be understood as the value predefined by the engine manufacturers, i.e. the one they seek approval for.

Crankshaft material is defined by its designation, mechanical properties (tensile strength, yield strength, elongation, reduction area at break and impact energy), its production process (cast, free form forged, continuous grain flow forged, or drop-forged) and the surface treatment applied to fillets or oil holes. These data can be found in the material specifications.

The crankshaft constructive structural shape, with all the relevant dimensions can be determined from the crankshaft workshop drawings submitted for approval. These data are defined in Figure 1 [1]. Shrink fits and oil bores require some additional dimensions to be defined, as described in [2].

$$\cos\beta = +\sqrt{1-\lambda^2 \cdot \sin^2\alpha} \tag{2}$$

Differentiating the equation (1) with respect to time t, taking into account that the crank rotational speed ω is considered constant we obtain:

$$\cos\beta \cdot \frac{d\beta}{dt} = \lambda \cdot \cos\alpha \cdot \frac{d\alpha}{dt} \tag{3}$$

Equation (3) gives the connecting rod angular velocity ω_{CR}:

$$\omega_{CR} = \frac{d\beta}{dt} = \frac{\lambda \cdot \cos\alpha}{\cos\beta} \cdot \frac{d\alpha}{dt} = \omega \cdot \frac{\lambda \cdot \cos\alpha}{\cos\beta} \tag{4}$$

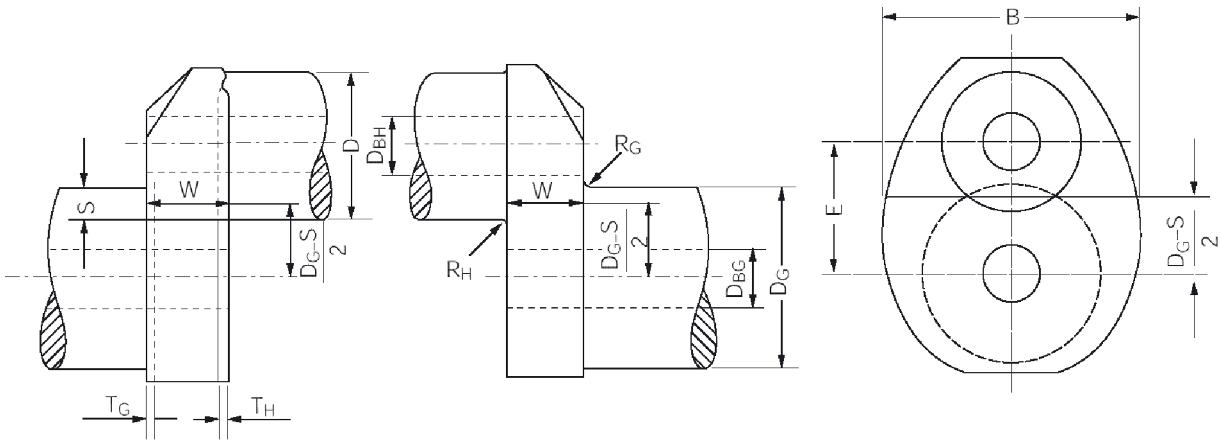

Figure 1 Crank dimensions necessary for the calculation
Slika 1. Izmjere osnog koljena neophodne za proračun

3 Methods of loading calculations

3.1 Kinematics of the single crank mechanism

The basic idea is to begin by analysing the connecting rod motion, instead of the usual start with the motion of the reciprocating parts. This approach will avoid both the unnecessarily complicated formulae as well as any approximations, often found in the literature describing the usual approach such as [6-10]. The schematics of the single crank mechanism model are given in Figure 2.

The connecting rod is in a composite motion, which consists of its centre-of-mass translation and its rotation about the centre-of-mass. The connecting rod angular displacement and the crank angular displacement are related as follows (see Figure 2):

$$\sin\beta = \lambda \cdot \sin\alpha \tag{1}$$

or:

Imposing constant engine speed (ω=const.) the connecting rod angular acceleration is obtained by differentiating the equation (4) with respect to time t, after a little bit of tidying up:

$$\varepsilon_{CR} = \frac{d^2\beta}{dt^2} = -\omega^2 \cdot \frac{\lambda \cdot \left(1-\lambda^2\right) \cdot \sin\alpha}{\cos^3\beta} \tag{5}$$

It is worth noting that the equations (1) to (5) are valid in any quadrant, since the absolute connecting rod angular displacement satisfies: $|\beta|$<90 º.

The characteristic points of the connecting rod are denoted R (rotational point – crank pin), S (connecting rod centre of mass) and T (translational point – the connection with the piston pin, or the crosshead), as shown in Figure 2. The chosen coordinate system is also shown in the same Figure. The coordinates of these characteristic points (x, y), their velocities (v_x, v_y) and accelerations (a_x, a_y) along the chosen coordinate axes, positive in the axes positive direction, may be calculated as presented in Table 1.

For comparison with the conventional usual approach, the piston displacement s_p (measured from the TDC), velocity v_p, and acceleration a_p (all positive downwards,

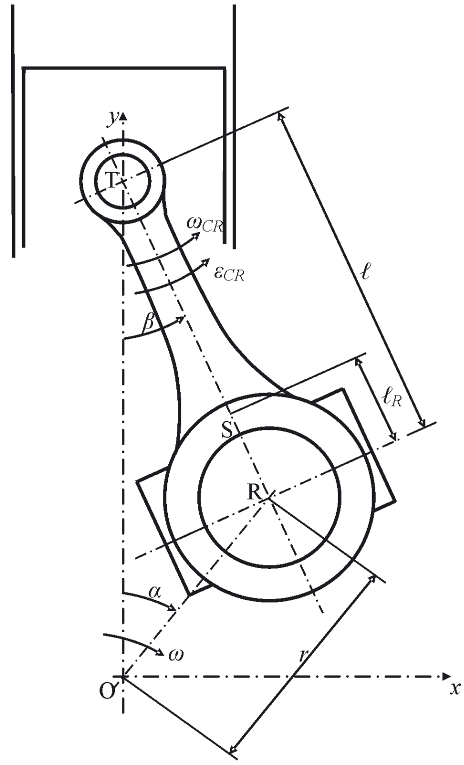

Figure 2 Schematics of a Diesel engine crank mechanism model

Slika 2. Shematski prikaz modela dizel motornog mehanizma

i.e. towards the crank axis of rotation) may be expressed in analytical form. From the values in Table 1:

$$s_p = r + \ell - y_T = r\cdot(1-\cos\alpha) + \ell\cdot(1-\cos\beta) \tag{6}$$

$$v_p = -v_{y,T} = r\omega\cdot\sin\alpha + \ell\cdot\omega_{CR}\cdot\sin\beta \tag{7}$$

$$a_p = -a_{y,T} = r\omega^2\cdot\cos\alpha + \ell\cdot\left(\varepsilon_{CR}\cdot\sin\beta + \omega_{CR}^2\cdot\cos\beta\right) \tag{8}$$

Table 1 Positions, velocities and accelerations of the connecting rod characteristic points

Tablica 1. Položaj, brzina i ubrzanje karakterističnih točaka ojnice

point:	**R**	**S**	**T**
x	$r\cdot\sin\alpha$	$(\ell-\ell_R)\cdot\sin\beta$	0
y	$r\cdot\cos\alpha$	$r\cdot\cos\alpha+\ell_R\cdot\cos\beta$	$r\cdot\cos\alpha+\ell\cdot\cos\beta$
v_x	$r\omega\cdot\cos\alpha$	$(\ell-\ell_R)\cdot\omega_{CR}\cdot\cos\beta$	0
v_y	$-r\omega\cdot\sin\alpha$	$-r\omega\cdot\sin\alpha-\ell_R\cdot\omega_{CR}\cdot\sin\beta$	$-r\omega\cdot\sin\alpha-\ell\cdot\omega_{CR}\cdot\sin\beta$
a_x	$-r\omega^2\cdot\sin\alpha$	$(\ell-\ell_R)\cdot(\varepsilon_{CR}\cos\beta-\omega_{CR}^2\sin\beta)$	0
a_y	$-r\omega^2\cdot\cos\alpha$	$-r\omega^2\cos\alpha-\ell_R\cdot(\varepsilon_{CR}\sin\beta+\omega_{CR}^2\cos\beta)$	$-r\omega^2\cos\alpha-\ell\cdot(\varepsilon_{CR}\sin\beta+\omega_{CR}^2\cos\beta)$

Note that the non-dimensional relations (i.e. the relative piston displacement s_p/s, the relative piston velocity v_p/v_m, as well as the relative piston acceleration a_p/g) are convenient to present the piston kinematics, especially in diagrams, in order to understand its behaviour. It is also important to note that all of the above-presented kinematic values are analytical, meaning that they do not contain any approximations, which are often found in literature in a form of series expansions or similar.

3.2 Dynamics of the single crank mechanism in rectangular coordinates

The analysis of the single crank mechanism loading also begins with the analysis of the forces acting on the connecting rod. These are external active forces, external passive forces (bearing reactions) and inertial forces (due to accelerations of masses, in accordance with the D'Alembert principle). The general situation of engine cylinder axis inclined from the vertical by angle φ_v as present in V-engines will be considered hereafter, so that the case of in-line engines (φ_v=0) may be considered as the special case.

The external active forces acting on the connecting rod are the gas forces acting on the piston and transferred to the connecting rod point T, as well as the weights of components (of masses m_{CR} and m_p) acting in the points S and T respectively. The gas forces originate from the known and given cylinder pressure vs. crank angle table values $p=p(\alpha)$, so called the expanded indicator diagram values. For every crank angle α the gas forces are determined as follows:

$$F_p = \frac{\pi\cdot d^2}{4}\cdot p(\alpha) \tag{9}$$

In order to present the $p(\alpha)$ values in form of the curve the non-dimensional presentation would be convenient in form of the relationship $p/p_{m,i}$ vs. crank angle α. The mean indicated pressure can be directly calculated as follows:

$$p_{m,i} = \frac{1}{s}\int p(x)\cdot dx =$$
$$= \frac{1}{2}\int_{\alpha_{\min}}^{\alpha_{\max}} p(\alpha)\left(\sin\alpha + \lambda\frac{\sin\alpha\cdot\cos\alpha}{\sqrt{1-\lambda^2\sin^2\alpha}}\right)\cdot d\alpha \tag{10}$$

The integrals in equation (10) can be numerically calculated precisely enough by means of the trapezoidal rule, since the $p(\alpha)$ values are discrete.

For the sake of good order it is important to point out here that, once the mechanical efficiency ratio η_{mech} is known, the mean effective pressure $p_{m,e}$ is determined as follows:

$$p_{m,e} = \eta_{mech}\cdot p_{m,i} \tag{11}$$

Mean indicated power and mean effective power per engine cylinder at the engine nominal speed are determined as follows:

$$P_{m,i} = \frac{\pi}{4} d^2 \cdot p_{m,i} \cdot \frac{v_m}{z} \tag{12}$$

$$P_{m,e} = \eta_{mech} \cdot P_{m,i} \tag{13}$$

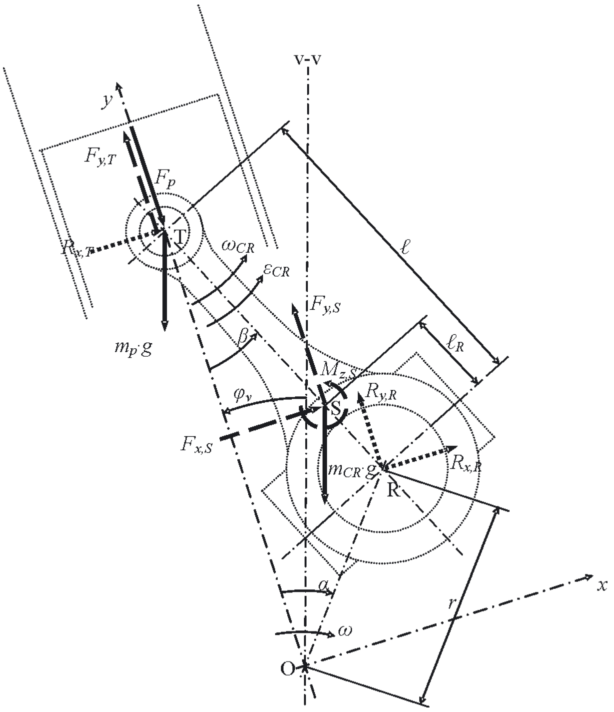

Figure 3 External active, external passive and inertial forces acting on the connecting rod

Slika 3. Vanjske aktivne, vanjske pasivne i inercijske sile koje djeluju na ojnicu

The external passive forces acting on the connecting rod are shown in Figure 3 (dotted vectors) together with the external active ones. They are the reactions in the connecting rod bearings. The reaction in point T acts in horizontal direction, whereas the reaction in point R is decomposed into the two components: horizontal and vertical.

The inertial forces acting on the connecting rod are also presented in Figure 3 with their values shown in Table 2. The inertia of the connecting rod around its centre of gravity J_{CR} is taken into account directly, based on the theory presented in [11]. Subdivision of the connected rod mass into two or three parts (e.g. in [3, 8] or [12]) showed to be unnecessary and is thus avoided. The J_{CR} is defined as follows:

$$J_{CR} = m_{CR} \cdot i_{CR}^2 \tag{14}$$

Positive directions of the inertial forces (and the inertial moment) are shown in Figure 3 and they act along the positive directions of the coordinate axes.

Since D'Alembert principle has been implemented the evaluation of the unknowns (external passive forces) transforms the dynamic problem to the static one. The unknown forces are determined from the three static equations (ΣM_R=0, ΣF_y=0 and ΣF_x=0), obtaining the final expressions:

$$\begin{aligned} R_{x,T} &= F_p \tan\beta - \frac{J_{CR}\varepsilon_{CR}}{\ell\cos\beta} + \\ &+ \left(m_{CR}\frac{\ell_R}{\ell} + m_p \right) g \frac{\sin(\beta + \varphi_v)}{\cos\beta} + \\ &+ m_{CR}\frac{\ell_R}{\ell}\left[a_{x,S} + a_{y,S}\tan\beta \right] + \\ &+ m_p \left(a_{x,T} + a_{y,T}\tan\beta \right) \end{aligned} \tag{15}$$

$$\begin{aligned} R_{x,R} &= -R_{x,T} + \left(m_{CR} + m_p \right) g \sin\varphi_v + \\ &+ m_{CR} a_{x,S} + m_p a_{x,T} \end{aligned} \tag{16}$$

$$\begin{aligned} R_{y,R} &= F_p + \left(m_{CR} + m_p \right) g \cos\varphi_v + \\ &+ m_{CR} a_{y,S} + m_p a_{y,T} \end{aligned} \tag{17}$$

Note that the values necessary to determine the reactive forces in equations (15) through (17) have been given by expressions (5), (9), (14) and those in Table 1. Once the reactive forces acting on the connecting rod have been obtained, they become the active forces, acting to the crank pin. The total forces acting on the crank reduced to the crank rotation axis (see Figure 4) are determined by rotating the coordinate axes by the angle α and by adding the crank self weight and the centrifugal force:

$$F_t = -R_{x,R}\cos\alpha + R_{y,R}\sin\alpha + m_W g \sin(\alpha - \varphi_v) \tag{18}$$

$$\begin{aligned} F_r &= R_{x,R}\sin\alpha + R_{y,R}\cos\alpha + \\ &+ m_W \left[g\cos(\alpha - \varphi_v) - r_W \omega^2 \right] \end{aligned} \tag{19}$$

$$\begin{aligned} M_t &= r \cdot \left(-R_{x,R}\cos\alpha + R_{y,R}\sin\alpha \right) + \\ &+ r_W m_W g \sin(\alpha - \varphi_v) \end{aligned} \tag{20}$$

The finally obtained forces acting on the crank as stated in equations (18) to (20) are the ones necessary for the fatigue calculations and torsional vibrations excitation evaluations.

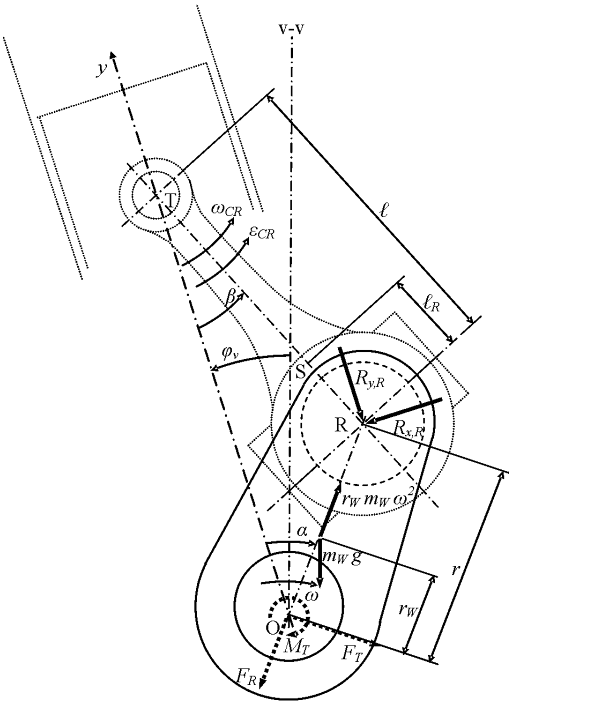

Figure 4 Forces acting on the crank web and their reduction to the axis of rotation

Slika 4. Sile koje djeluju na osno koljeno i njihova redukcija na os vrtnje

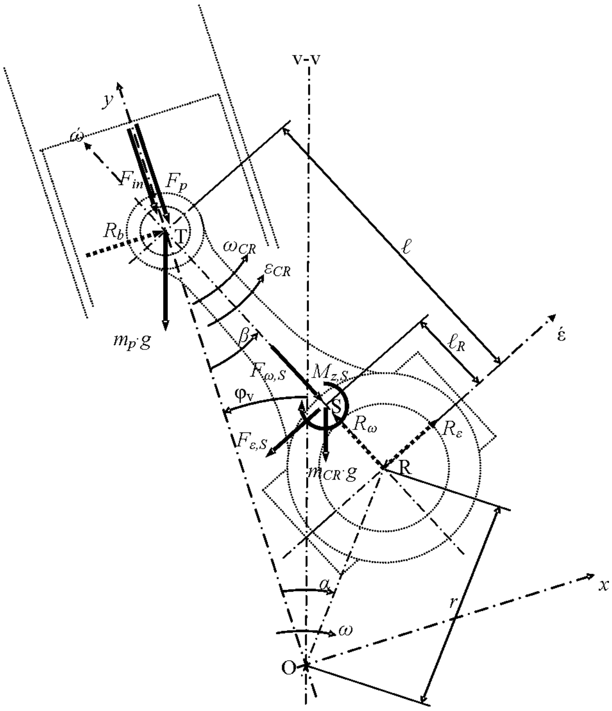

Figure 5 Forces acting on the connecting rod expressed in natural coordinates

Slika 5. Sile koje djeluju na ojnicu izražene u prirodnim koordinatama

3.3 Dynamics of the single crank mechanism in natural coordinates

An alternative approach using natural coordinates in order to determine forces acting on the crank will also be presented, as it has been found useful to verify the correctness of the expressions in rectangular coordinates. The connecting rod motion is composed of the translation expressed by the acceleration a_p (equation 8) and the rotation expressed by the angular velocity ω_{CR} (equation 4) and angular acceleration ε_{CR} (equation 5). External active, external passive and inertial forces acting on the connecting rod are shown in Figure 5, all together, showing the chosen positive directions of the natural coordinates (denoted as έ and ώ).

Table 3 shows the kinematic and dynamic quantities necessary for the evaluation of the unknown reactive forces R_b, R_ε and R_ω.

By means of D'Alembert principle the unknown reaction forces are determined from the three static equations (ΣM_R=0, ΣM_T=0 and ΣF_ω=0), providing finally: (Note: piston acceleration a_p is determined by equation (8), and the inertial force $F_{in}=m_p \cdot a_p$)

Table 2 Weights and inertial forces acting on the connecting rod in the characteristic points

Tablica 2. Težine i inercijske sile na ojnicu u karakterističnim točkama

point:	R	S	T
G_x	0	$-m_{CR} g \cdot \cos\varphi_v$	$-m_p g \cdot \cos\varphi_v$
G_y	0	$-m_{CR} g \cdot \sin\varphi_v$	$-m_p g \cdot \sin\varphi_v$
F_x	0	$-m_{CR} \cdot a_{x,S}$	$-m_p g \cdot a_{x,T}$
F_y	0	$-m_{CR} \cdot a_{y,S}$	$-m_p g \cdot a_{y,T}$
M_z	0	$-J_{CR} \cdot \varepsilon_{CR}$	0

Table 3 Required kinematic and dynamic values in the characteristic points R and S of the connecting rod

Tablica 3. Potrebne kinematičke i dinamičke veličine u karakterističnim točkama R i S ojnice

point:	R	S
a_ε	$-a_p \cdot \sin\beta + \ell \cdot \varepsilon_{CR}$	$-a_p \cdot \sin\beta + (\ell - \ell_R) \cdot \varepsilon_{CR}$
a_ω	$-a_p \cdot \cos\beta + \ell \cdot \omega_{CR}^2$	$-a_p \cdot \cos\beta + (\ell - \ell_R) \cdot \omega_{CR}^2$
F_ε	0	$m_{CR} \cdot a_{\varepsilon,S}$
F_ω	0	$m_{CR} \cdot a_{\omega,S}$
M_z	0	$J_{CR} \cdot \varepsilon_{CR}$

$$R_b = \left(F_p + F_{in}\right)\cdot \tan\beta + \\ +\left(m_{CR}\frac{\ell_R}{\ell} + m_p\right) g \frac{\sin\left(\beta+\varphi_v\right)}{\cos\beta} - \\ -\frac{J_{CR}\varepsilon_{CR}}{\ell\cos\beta} + \frac{F_{\varepsilon,S}}{\cos\beta}\frac{\ell_R}{\ell} \tag{21}$$

$$R_\varepsilon = m_{CR}\left(1-\frac{\ell_R}{\ell}\right) g \sin\left(\beta+\varphi_v\right) + \\ +\frac{J_{CR}\varepsilon_{CR}}{\ell} + F_{\varepsilon,S}\cdot\left(1-\frac{\ell_R}{\ell}\right) \tag{22}$$

$$R_\omega = \left(F_p + F_{in}\right)\cos\beta + \\ +\left(m_{CR}+m_p\right) g\cos\left(\beta+\varphi_v\right) + \\ +F_b\sin\beta + F_{\omega,S} \tag{23}$$

The total forces acting on the crank are obtained by rotating the coordinate system around its origin for the angle $\alpha+\beta$ adding the crank self weight and the centrifugal force:

$$F_t = R_\omega \sin\left(\alpha+\beta\right) - R_\varepsilon\cos\left(\alpha+\beta\right) + \\ +m_W g\sin(\alpha-\varphi_v) \tag{24}$$

$$F_r = R_\omega\cos\left(\alpha+\beta\right) + R_\varepsilon\sin\left(\alpha+\beta\right) + \\ +m_W\left[g\cos(\alpha-\varphi_v) - r_W\omega^2\right] \tag{25}$$

Results obtained by equations (24) and (25) will be the same as the ones provided by equations (18) and (19).

3.4 Trigonometric approximation of engine operating parameters

IACS UR M53 (2004) prescribes: "the maximum and minimum torques are to be ascertained for every mass point … by means of a harmonic synthesis of the forced vibrations from the 1st order up to and including the 15th order for 2-stroke cycle engines and from the 0,5th order up to and including the 12th order for 4-stroke cycle engines" [2]. However, the document [2] itself does not provide a word describing how to do it. So, the next expressions will describe the trigonometric approximation of the cylinder pressure *p(α)*, as well as the tangential force on the crank pin F_t, as the essential step in the quoted IACS requirement. Note that the requirement actually calls for the torsional vibrations calculation, in which the trigonometric approximations of either of the mentioned parameters are needed.

The function *p(α)*, given by values of *p* in discrete points α_1, α_2, ... α_m is to be approximated by the trigonometric sum in accordance with the following expression:

$$p_N(\alpha) = \frac{A_0}{2} + \sum_{k=1}^{N} A_k\cos\frac{k\alpha}{z/2} + B_k\sin\frac{k\alpha}{z/2} \tag{26}$$

The trigonometric approximation gives the best results when the coefficients A_k and B_k are taken as Fourier coefficients [12]:

$$A_k = \frac{1}{\pi\cdot z/2}\int_{\alpha_{\min}}^{\alpha_{\max}} p(\alpha)\cos\frac{k\alpha}{z/2}d\alpha\ ; \\ k = 0,\ 1,\ 2,\ \ldots, N \tag{27}$$

$$B_k = \frac{1}{\pi\cdot z/2}\int_{\alpha_{\min}}^{\alpha_{\max}} p(\alpha)\sin\frac{k\alpha}{z/2}d\alpha\ ; \\ k = 0,\ 1,\ 2,\ \ldots, N \tag{28}$$

Note that: $\alpha_{\min,\max}=\pm\pi\cdot z/2$ (i.e. ±180º for two-stroke and ±360º for four-stroke engines). Since the function *p(α)* is given in discrete points, the integrals in (27) and (28) shall be calculated numerically, e.g. by means of the trapezoidal rule:

$$A_k = \frac{1}{\pi\cdot z}\sum_{i=2}^{m}\left(\alpha_i - \alpha_{i-1}\right)\cdot \\ \cdot\left(p_i\cos\frac{k\alpha_i}{z/2} + p_{i-1}\cos\frac{k\alpha_{i-1}}{z/2}\right) \tag{29}$$

$$B_k = \frac{1}{\pi\cdot z}\sum_{i=2}^{m}\left(\alpha_i - \alpha_{i-1}\right)\cdot \\ \cdot\left(p_i\sin\frac{k\alpha_i}{z/2} + p_{i-1}\sin\frac{k\alpha_{i-1}}{z/2}\right) \tag{30}$$

Quality of numerical approximation can be judged, if necessary, by either of the three following numerical errors:

$$\delta_{\max} = \max\left\{\ \left|p_i - p_{i,appr}\right|\ ;\ i=1,\ 2,\ \ldots,\ m\right\} \tag{31}$$

$$\delta_m = \frac{1}{m}\sum_{i=1}^{m}\left|p_i - p_{i,appr}\right| \tag{32}$$

$$\delta_s = \frac{1}{m}\sqrt{\sum_{i=1}^{m}\left(p_i - p_{i,appr}\right)^2} \tag{33}$$

In case of the torsional vibrations excitation evaluation the approximation of the tangential force acting on the crank F_t is needed. It is obtained following the same procedure by substituting F_t values into equations (26) to (33).

Trigonometric approximations are of utmost importance in case of V-engines, because the distance among the two neighbouring angles α_i and α_{i-1} might be either not uniform or not a common multiplier of the V-angle $2\varphi_v$.

3.5 Influence of the cylinder ignition sequence

The trigonometric approximation presented by equation (26) in terms of crank angles α may be expressed in terms of time, substituting:

$$\alpha = \omega \cdot t + \theta \tag{34}$$

If the values $p(\alpha)$ are given for the phase shift θ=0º and the trigonometric approximation coefficients A_k and B_k are obtained by expressions (29) and (30) in this case, then the coefficients A'_k and B'_k for the phase shift θ are obtained by the rotations:

$$A'_k = A_k \cos\frac{k\theta}{z/2} + B_k \sin\frac{k\theta}{z/2} \tag{35}$$

$$B'_k = -A_k \sin\frac{k\theta}{z/2} + B_k \cos\frac{k\theta}{z/2} \tag{36}$$

Note that the equation (26) applies directly to the shifted-in-phase case ($\theta \neq 0$º), substituting the coefficients A_k and B_k with the coefficients A'_k and B'_k.

3.6 Calculation of stress concentration factors and operational stresses

Fatigue strength calculation in [2] is based upon the nominal stress approach. This means that the equivalent stresses are calculated from the nominal stress components increased by stress concentration factors. They are to be compared with the fatigue strength, which is dependent upon the crankshaft manufacturing process, surface treatment, size (diameter) and fillet radii.

The calculation of stresses, stress concentration factors and fatigue strength is described in IACS UR M53 (2004) [2] with enough details for its practical application, so it will be only briefly described hereafter, stating only the essential expressions. However, the calculation of the bending moments, not entirely described in [2], deserves some additional care, since it differs for the in-line and for the V-engines.

Implementing the nominal stress approach, alternating bending moments M_{BrN} and alternating shear forces Q_{rN} decisive for fatigue strength are calculated for the statically determined system composed of a single crankthrow supported in the centre of adjacent main journals [2]. They are determined from the crank journal reaction forces R_r, dependent upon the total radial forces F_r acting on the crank. In the case of in-line-type engines R_r=F_r/2, whereas in the case of V-type engines the crank journal reaction forces R_{rA} and R_{rB} (adjacent to the connecting rods belonging to the cylinder banks A and B, respectively) are:

$$R_{rA\,\mathrm{min/max}} = \mathrm{min/max}\begin{Bmatrix} F_{rA}\left(1 - L_2/L_3\right) + \\ +F_{rB} \cdot L_2/L_3 \end{Bmatrix} \tag{37}$$

$$R_{rB\,\mathrm{min/max}} = \mathrm{min/max}\begin{Bmatrix} F_{rA} \cdot L_2/L_3 + \\ +F_{rB}\left(1 - L_2/L_3\right) \end{Bmatrix} \tag{38}$$

For in-line-type engines alternating shear force in the centre of the solid web is:

$$Q_{rN} = \left(F_{r\,\max} - F_{r\,\min}\right)/4 \tag{39}$$

and for V-type engines:

$$Q_{rN} = \max\begin{Bmatrix} \left(R_{rA\,\max} - R_{rA\,\min}\right)/2; \\ \left(R_{rB\,\max} - R_{rB\,\min}\right)/2 \end{Bmatrix} \tag{40}$$

For both engine types alternating bending moment in the centre of the solid web is:

$$M_{BrN} = Q_{rN} \cdot L_1 \tag{41}$$

Nominal alternating bending stresses in the centre of the solid web (of thickness W and width B) [2]:

$$\sigma_{BFN} = K_e \frac{M_{BrN}}{B \cdot W^2/6} \tag{42}$$

Nominal alternating compressive stresses [2]:

$$\sigma_{QFN} = K_e \frac{Q_{rN}}{B \cdot W} \tag{43}$$

Nominal alternating torsional stress τ_N is the value specified by the engine manufacturers', as the one they inquire approval for. In each complete (propulsion or auxiliary) installation, torsional vibration calculation is to prove that this value is not exceeded in the engine crankshaft.

Stress concentration factors for either the crank pin fillet in bending and torsion (α_B, α_T), or the journal fillet in bending, compression and torsion (β_B, β_Q, β_T) are explicitly defined in [1, 2] dependent upon crankshaft dimensions (Figure 1) and need not be repeated here.

Equivalent alternating stress in the crankpin fillet [1, 2]:

$$\sigma_V = \sqrt{\left(\alpha_B \cdot \sigma_{BFN} + \sigma_{add}\right)^2 + 3 \cdot \left(\alpha_T \cdot \tau_N\right)^2} \tag{43}$$

Equivalent alternating stress in the crank journal fillet [1, 2]:

$$\sigma_V = \sqrt{\left(\beta_B \cdot \sigma_{BFN} + \beta_Q \cdot \sigma_{QFN} + \sigma_{add}\right)^2 + 3 \cdot \left(\beta_T \cdot \tau_N\right)^2} \quad (44)$$

3.7 Calculation of fatigue strength and acceptability criteria

Fatigue strength is that value of equivalent alternating stress (Von Mises stress) which the crankshaft can permanently withstand at the most highly stressed points (crankpin fillets and journal fillets). It is dependent upon manufacturing process and surface treatment (factor K), diameter (d_X), material tensile strength (σ_B) and fillet radius (r_X). The following formula may be equally applied to determine fatigue strength both in the crankpin fillet and in the journal fillet [1, 2]:

$$\sigma_{DW} = K\left(0,42\sigma_B + 39,3\right) \cdot \cdot \begin{pmatrix} 0,264 + 1,073 \cdot d_X^{-0,2} + \\ + \dfrac{785 - \sigma_B}{4900} + \dfrac{196}{\sigma_B} \cdot \sqrt{\dfrac{1}{r_X}} \end{pmatrix} \quad (45)$$

Acceptability criterion is the ratio of fatigue strength and equivalent alternating stress in crankpin fillets and journal fillets. The crankshaft is adequately dimensioned when the following expression is satisfied [1, 2]:

$$\frac{\sigma_{DW}}{\sigma_V} \geq 1,15 \quad (46)$$

Additional acceptability criteria in [2], comprising fatigue strength in oil bores and safety of shrink fit in the semi-built crankshafts are beyond the scope of this paper.

4 Example and a brief discussion of results

The previously presented theory has been implemented in the program *S02Crank*, developed in the Croatian Register of Shipping by means of MS Excel/VBA to be used in the process of Diesel engines type approval. Compared with the previous program described in [3, 4] the present one had to be written from scratch because it fully implements the previously presented theory.

To illustrate the implementation of the program an example of crankshaft fatigue strength calculation for an actual engine, type approved in the past by the Croatian Register of Shipping (CRS), has been selected and shown below. This is an in-line type low-speed two-stroke crosshead turbocharged marine Diesel engine. Engine input data and calculation results are shown in Table 4. Harmonic analysis components are shown in Table 5, whereas the crankshaft fatigue strength input data and calculation results are shown in Table 6. Graphic presentation of non-dimensional kinematic as well as dynamic values is shown in Figures 6 to 9. Values obtained by harmonic analysis components are shown in Figure 6 for cylinder pressures and in Figure 9 for tangential forces, together with the harmonic analysis input data, proving practically the quality of trigonometric approximation.

The obtained results may be briefly discussed as follows. It has been practically proved that the subdivision of connecting rod masses into two- or three parts is unnecessary – the mass moment of inertia around its centre of gravity is to be implemented directly. An interesting fact obvious from non-dimensional graphs of kinematic values is that acceleration of reciprocating components obtains rather high values regardless of the slow engine speed. Consequently the influence of inertia forces to the total forces is significant: roughly 20 per

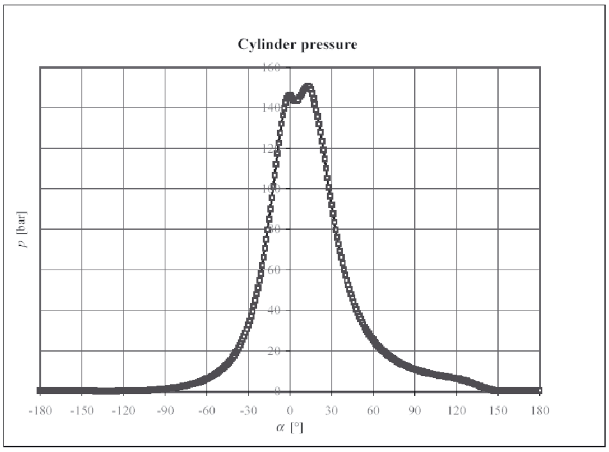

Figure 6 Cylinder pressures vs. crank angle curve

Slika 6. Krivulja tlakova u cilindru ovisno o kutu osnog koljena

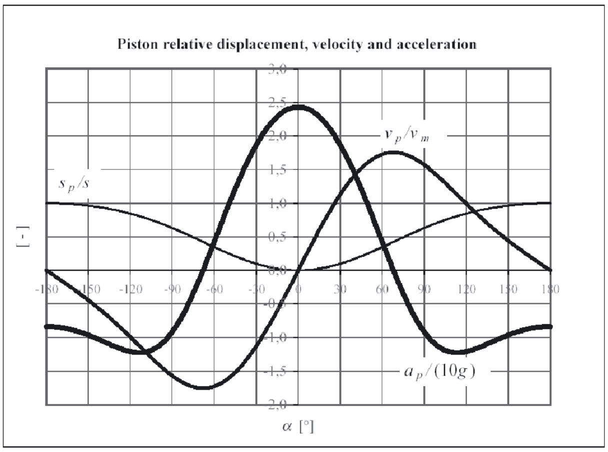

Figure 7 Kinematic values describing piston movement

Slika 7. Kinematičke veličine koje opisuju gibanje stapa

Table 4 Engine general input data and calculation results for the selected calculation example

Tablica 4. Opći ulazni podaci o motoru i rezultati za izabrani proračunski primjer

CROATIAN REGISTER OF SHIPPING								
CRANKSHAFT CALCULATIONS FOR INTERNAL COMBUSTION ENGINES								
Program: **S02CrankL.xls/vba**					Ver.: **2.0**(21 MAR 2007)			
Engine manufacturer:			N/A		Type:	**(common rail)**		
ENGINE INPUT DATA			α	*p*	α	*p*	α	*p*
cycle=	2		[°]	[bar]	[°]	[bar]	[°]	[bar]
d=	600	mm	-180,0	0,5	1,0	145,7	12,0	150,2
r=	1125	mm	-179,0	0,5	2,0	144,5	13,0	150,7
l=	2300	mm	-178,0	0,5	3,0	143,6	14,0	150,4
n=	114	rpm	-177,0	0,5	4,0	143,3	15,0	149,0
m_p=	3525	kg	...	...	5,0	143,5	16,0	147,0
m_{CR}=	2084	kg	-5,0	136,2	6,0	144,2	17,0	144,6
ℓ_R=	1192	mm	-4,0	139,7	7,0	145,3	18,0	141,9
i_{CR}=	1005	mm	-3,0	142,5	8,0	146,4	...	...
m_W=	10074	kg	-2,0	144,6	9,0	147,5	178,0	0,5
r_W=	326	mm	-1,0	145,8	10,0	148,5	179,0	0,5
g=	9,80665	m/s^2	0,0	146,2	11,0	149,4	180,0	0,5
η_m=	0,95							
ENGINE OUTPUT RESULTS								
s=	2.250,00	mm	p_{mi}=	20,14	bar			
v_m=	8,55	m/s	P_i=	2.434,37	kW/cyl			
ω=	11,94	rad/s	p_{me}=	19,13	bar			
λ=	0,49		P_e=	2.312,65	kW/cyl			
	min	*max*	*mean*	*amp*				
F_t=	-1 012,74	1 100,63	43,94	**1 056,68**	kN			
F_r=	-780,77	2 964,93	1 092,08	**1 872,85**	kN			
T_t=	-1 139,33	1 238,21	49,44	1 188,77	kNm			

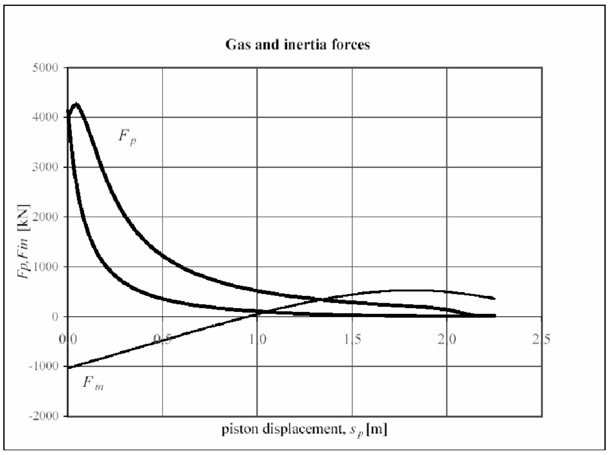

Figure 8 Gas and inertia forces on the piston

Slika 8. Sile uslijed tlakova i inercijske sile na stap

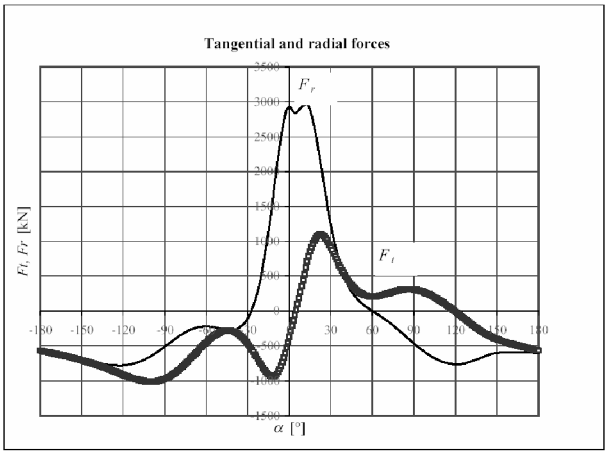

Figure 9 Forces acting on the crank pin

Slika 9. Sile na osno koljeno

Table 5 Harmonic analysis results

Tablica 5. Rezultati harmonijske analize

HARMONIC ANALYSIS OF ENGINE PARAMETERS									
	pressures [bar]		tangential forces [kN]						
k	*A(k)*	*B(k)*	*A(k)*	*B(k)*	*angle*	*p*	*p_appr*	*Ft*	*Ft_appr*
0	53,70889	0	-573,569	0	[º]	[bar]	[bar]	[kN]	[kN]
1	44,16721	10,15393	314,196	581,2161	-180	0,5	0,517	-566,84	-566,74
2	29,5867	10,46085	3,068852	128,7191	-179	0,5	0,513	-571,15	-571,138
3	19,67049	8,575934	-66,3442	85,24632	-178	0,5	0,499	-575,43	-575,516
4	11,86588	7,316912	-84,0828	225,8604	-177	0,5	0,487	-579,70	-579,815
5	6,50853	4,605767	-87,3935	194,3425	-176	0,5	0,485	-583,94	-584,016
6	3,708329	2,880035	-68,2304	113,997	-175	0,5	0,494	-588,18	-588,159
7	1,51771	1,522138	-53,8739	65,08343	-174	0,5	0,507	-592,42	-592,314
8	0,532002	0,262903	-37,7809	32,70408	-173	0,5	0,516	-596,65	-596,54
9	0,101242	-0,27106	-22,8812	16,09423	-172	0,5	0,513	-600,89	-600,845
10	-0,21301	-0,75088	-14,0546	5,771426	-171	0,5	0,501	-605,13	-605,188
11	-0,15828	-0,96952	-5,69383	-0,6648	-170	0,5	0,488	-609,40	-609,508
12	-0,13472	-0,98461	-0,60722	-2,30816	-169	0,5	0,483	-613,68	-613,77
13	-0,06032	-0,98855	2,388216	-3,11232	-168	0,5	0,49	-617,98	-617,992
14	0,027134	-0,8684	4,238898	-2,67201	-167	0,5	0,503	-622,32	-622,237
15	0,067627	-0,77401	4,700385	-1,81764	-166	0,5	0,515	-626,68	-626,571
16	0,111533	-0,64686	4,73419	-1,0872	-165	0,5	0,517	-631,09	-631,023
17	0,118264	-0,54326	4,238246	-0,52416	-164	0,5	0,507	-635,54	-635,566
18	0,135956	-0,44754	3,681789	-0,08416	-163	0,5	0,493	-640,04	-640,14
19	0,12164	-0,36559	2,944671	0,185971	-162	0,5	0,482	-644,60	-644,697
20	0,123742	-0,30903	2,32708	0,191363	-161	0,5	0,483	-649,21	-649,242
21	0,115614	-0,25729	1,935222	0,30858	-160	0,5	0,496	-653,89	-653,83
22	0,106611	-0,21623	1,583933	0,274435	-159	0,5	0,512	-658,63	-658,534
23	0,102767	-0,18372	1,366563	0,215336	-158	0,5	0,523	-663,45	-663,391
24	0,097051	-0,15074	1,176059	0,203672	-157	0,5	0,519	-668,35	-668,376

cent for the crank angles corresponding to the highest cylinder pressure. The acceptability factors of 1,80 for the crank pin fillet and 1,39 for the journal fillet are rather high compared with the required one of at least 1,15. This proves that the engine crankshaft has been correctly shaped and dimensioned from the point of its classification and that the type approval process was allowed to proceed to the next step: testing of the engine prototype.

5 Conclusive remarks

The goal of this paper is to develop the systematic procedure that relies on IACS UR M53 (2004) [2] *Calculation of Crankshafts for Internal Combustion Engines*. This procedure aims to enable a uniform and straightforward evaluation of crankshaft design and calculation of its fatigue strength for marine Diesel engines, as the part of engine type approval.

Table 6 Crankshaft input data and calculation results for the selected calculation example

Tablica 6. Ulazni podaci o koljenastom vratilu i rezultati za izabrani proračunski primjer

CRANKSHAFT INPUT DATA								
concept=	1	(crosshead)						
K=	1							
Material	crank web			journal				
type=	forged steel		*type*=	forged steel				
σ_B=	590	N/mm^2	σ_G=	590	N/mm^2			
σ_S=	370	N/mm^2	$\rho_{G=}$	7850	kg/m^3			
$\rho_{W=}$	7850	kg/m^3						
Dimensions	crankpin			journal			web / axial length	
D_H=	730	mm	D_G=	730	mm	W=	256	mm
D_{BH}=	0	mm	D_{BG}=	115	mm	B=	1160	mm
R_H=	55	mm	R_G=	11	mm	L_1=	267	mm
T_H=	55	mm	T_G=	0	mm	L_2=	520	mm
						L_3=	1040	mm
Loading			**Acceptability criteria**					
τ_{Nmax}=	28,5	N/mm^2	$(\sigma_{DW}/\sigma_V)_{min}$=	1,15				
CRANKSHAFT FATIGUE STRENGTH CALCULATION RESULTS								
Stress concentration factors in fillets								
		bending		shearing		torsion		
crankpin	α_B=	**2,538**	α_Q=	N/A	α_T=	**1,474**		
journal	β_B=	**4,689**	β_Q=	**0,578**	β_T=	**1,924**		
Internal forces			**Nominal stresses**					
K_e=	0,8		σ_{BN}=	15,79	N/mm^2			
M_{BN}=	250 025,87	Nm	σ_{QN}=	2,52	N/mm^2			
Q_N=	936 426,47	N						
Crankpin fillet			Journal fillet					
σ_{BH}=	40,07	N/mm^2	σ_{BG}=	75,49	N/mm^2			
σ_{Hadd}=	30,00	N/mm^2	σ_{Gadd}=	30,00	N/mm^2			
τ_H=	41,99	N/mm^2	τ_G=	54,85	N/mm^2			
σ_{VH}=	100,99	N/mm^2	σ_{VG}=	141,96	N/mm^2			
σ_{DWH}=	182,49	N/mm^2	σ_{DWG}=	198,38	N/mm^2			
$(\sigma_{DW}/\sigma_V)_H$=	**1,81**		$(\sigma_{DW}/\sigma_V)_G$=	**1,40**				
criteria:	satisfied		criteria:	satisfied				

The basic task of the paper is to systematise and present all the missing parts in the document [2] itself, i.e. kinematics of the single crank mechanism, connecting rod dynamics in rectangular and natural coordinates, as well as trigonometric approximation of the cylinder pressures and crank tangential forces. All these are unavoidable for the calculation of crankshaft fatigue strength in the crank pin and crank journal fillets. The remaining part of the calculation procedure (from the nominal stresses and the stress concentration factors to the acceptability criteria) is comprehensively covered by [2]. The further task is to develop and implement a computer program for these calculations starting with the data normally expected to be available at the beginning of the engine type approval process: the reciprocating mechanism dimensions, its mass properties, nominal rotational speed and the cylinder pressure vs. crank angle curve.

The contribution of this paper would be the formulation of the unified procedure for kinematic and dynamic analysis of the reciprocating mechanism starting with the connecting rod motion instead of the piston motion. Closed solutions are preferred and provided in simple terms – avoiding expansion of the particular values into series. Moreover, the mass properties of the connecting rod are to be taken into calculation directly, based upon dynamics of rigid body planar motion, instead of its subdivision into two (reciprocating and rotational) or even three parts, the latter approach being usually found in textbooks. This aims to be the additional contribution.

The reciprocating mechanism kinematics has been presented firstly with all the necessary details. The suggestion is given how to express displacements, velocities, accelerations and pressures in a non-dimensional form. The dynamics of the mechanism in rectangular coordinates is completed further on with its equivalent presentation in natural coordinates with the aim to enable mutual review of the presented equations – results shall be identical either way. The presented approach is general enough to cover both in-line type and V-type engines simultaneously. Trigonometric approximations (Fourier harmonic analysis) of pressures and tangential forces have been presented with all the necessary details, as these approximations are inevitable in calculations of torsional vibrations and fatigue strength of V-type engines.

The computer program *S02Crank* is developed and tested in the globally widespread tool: MS Excel/VBA (Visual Basic for Applications) to implement all of the presented theory. An application of the program to an actual engine, on the basis of the input data provided by the manufacturer for the engine type approval is also shown and briefly discussed.

Explanation of experimental approach in crankshaft fatigue strength validation, which is sometimes used by certain four-stroke high-speed engine manufacturers, is not presented here as it lies beyond the scope of this paper.

Implementation of the presented approach in the calculation of torsional vibrations for propulsion and auxiliary systems comprising marine Diesel engines, with the same basic idea of starting by the cylinder pressures table within the calculation of excitation data, will be the matter of future development and work.

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