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NUMERICAL PROCEDURE FOR SHIP HYDROELASTIC ANALYSIS

I. SENJANOVIĆ[*], S. TOMAŠEVIĆ[*], N. VLADIMIR[*] AND Š. MALENICA[†]

[*] University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture
Ivana Lucica 5, 10000 Zagreb, CROATIA, e-mail: ivo.senjanovic@fsb.hr
[†] Bureau Veritas, 67/71 Boulevard du Châteu, 92200 Neuilly-sur-Seine, FRANCE

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Summary. Mathematical model for ship hydroelastic analysis, comprised of structural, hydrostatic and hydrodynamic models, is described. The modal superposition method is used and ship natural modes are determined for the sophisticated beam model based on the advanced thin-walled girder theory. The consistent restoring stiffness is formulated. Application of the numerical procedure is illustrated in the case of a large container ship.

1 INTRODUCTION

Large container ships are quite flexible concerning torsion and structural natural frequencies can fall into the range of encounter frequencies[1]. Therefore, hydroelastic response becomes very important issue for ship safety. The methodology of hydroelastic analysis includes the definition of the structural model, ship and cargo mass distributions, and geometrical model of ship surface. First, dry natural vibrations are calculated, and then modal hydrostatic stiffness, added mass, damping and wave load are determined. Finally, wet natural vibrations, as well as the transfer functions (RAO – response amplitude operator) for determining ship structural response to wave excitation, are obtained[2].

2 BEAM STRUCTURAL MODEL

The hydroelastic analysis can be performed by coupling 1D or 3D FEM structural model with 3D hydrodynamic model based on the radiation-diffraction theory[3]. The former hydroelastic model is more rational for preliminary design stage while the latter is used for the final strength analysis.

The beam model can give quite accurate results if it is based on the advanced thin-walled girder theory, i.e. by taking shear influence on bending and torsion, and stiffness contribution of transverse bulkheads into account in a reliable way. Total beam deflection and twist angle consist of pure bending and torsion, respectively, and shear contribution[4]

$$w = w_b + w_s = w_b - \frac{EI_b}{GA_s}\frac{\mathrm{d}^2 w_b}{\mathrm{d}x^2}, \qquad \psi = \psi_t + \psi_s = \psi_t - \frac{EI_w}{GA_s}\frac{\mathrm{d}^2 \psi_t}{\mathrm{d}x^2}, \tag{1}$$

where I_b is moment of inertia of cross-section, A_s is shear area, I_w is warping modulus and I_s is shear inertia modulus. We see that there is an analogy between bending and torsion

$$A_s = \frac{Q^2}{\int_A \tau_Q^2 \mathrm{d}A}, \quad I_s = \frac{T_w^2}{\int_A \tau_w^2 \mathrm{d}A}, \tag{2}$$

where Q and T_w are shear force and torque due to restrained warping, and τ_Q and τ_w are corresponding shear stresses, respectively.

The effect of large number of transverse watertight and support bulkheads can be incorporated into the hull torsional stiffness[5]

$$I_t^* = \left[1 + \frac{a}{l_1} + \frac{4(1+\nu)C}{I_t l_0}\right] I_t, \quad C = \frac{U}{E\psi_t'^2}, \tag{3}$$

where a is the web height of bulkhead girders, l_0 is the bulkhead spacing, $l_1 = l_0 - a$ is the net length, C is the energy coefficient, and U is the bulkhead grillage and stool strain energy due to warping of cross-section. Warping shape function can be assumed in the following form

$$\bar{u}(y,z) = -y\left\{(z-d) + \left[1 - \left(\frac{y}{b}\right)^2\right]\frac{z^2}{H}\left(2 - \frac{z}{H}\right)\right\}, \quad u(y,z) = \bar{u}(y,z)\psi_t', \tag{4}$$

where H is the ship height, b is one half of bulkhead breadth, d is the distance of warping centre from double bottom centroid, while y and z are transverse and vertical coordinates, respectively.

The governing matrix equation of dry natural vibrations in a FEM analysis yields

$$\left(\mathbf{K} - \Omega^2\mathbf{M}\right)\delta = \mathbf{0}, \tag{5}$$

where **K** is stiffness matrix, **M** is mass matrix, Ω is dry natural frequency and $\boldsymbol{\delta}$ is dry natural mode. As solution of the eigenvalue problem (5), Ω_i and $\boldsymbol{\delta}_i$ are obtained for each the i-th dry mode, where $i = 1,2...N$, N is total number of degrees of freedom. The first six natural frequencies Ω_i are zero with corresponding eigenvectors representing the rigid body modes.

If 1D analysis is applied, the beam modes are spread to the ship wetted surface using the expressions for vertical and coupled horizontal and torsional vibrations, respectively[4]

$$\mathbf{h}_i = -\frac{\mathrm{d}w_{vi}}{\mathrm{d}x}(z - z_N)\mathbf{i} + w_{vi}\mathbf{k}, \quad \mathbf{h}_i = \left(-\frac{\mathrm{d}w_{hi}}{\mathrm{d}x}y + \frac{\mathrm{d}\psi_i}{\mathrm{d}x}\bar{u}\right)\mathbf{i} + \left[w_{hi} + \psi_i(z - z_S)\right]\mathbf{j} - \psi_i y\mathbf{k}, \tag{6}$$

where w is hull deflection, ψ is twist angle, y and z are coordinates of the point on ship surface, and z_N and z_S are coordinates of centroid and shear centre, respectively.

3 HYDRODYNAMIC MODEL

The coupling procedure does not depend on the used hydrodynamic model, and is therefore described here for the zero speed case, as the simplest one. Harmonic hydroelastic problem is considered in frequency domain and therefore we operate with amplitudes of forces and displacements. In order to perform the coupling of structural and hydrodynamic models, it is necessary to express the external pressure forces in a convenient manner[6]. First, the total hydrodynamic force F^h has to be split into two parts: the first part F^R depending on the structural deformations, and the second one F^{DI} representing the pure excitation. Furthermore, the modal superposition method can be used. Vector of the wetted surface deformations $\mathbf{H}(x, y, z)$ can be presented as a series of dry natural modes $\mathbf{h}_i(x, y, z)$.

The potential theory assumptions are adopted for the hydrodynamic part of the problem. Within this theory, the total velocity potential φ, in the case of no forward speed, is defined with the Laplace differential equation and the given boundary values. Furthermore, the linear wave theory enables the following decomposition of the total potential[1]

$$\varphi = \varphi_I + \varphi_D - i\omega \sum_{j=1}^{N} \xi_j \varphi_{Rj} , \qquad \varphi_I = -i \frac{gA}{\omega} \mathrm{e}^{\nu(z+ix)} , \tag{7}$$

where φ_I is incident wave potential, φ_D is diffraction potential, φ_{Rj} is radiation potential and A and ω represent wave amplitude and frequency respectively.

Once the potentials are determined, the modal hydrodynamic forces are calculated by pressure work integration over the wetted surface, S. The total linearised pressure can be found from Bernoulli's equation

$$p = i\omega\rho\varphi - \rho g z . \tag{8}$$

First, the term associated with the velocity potential φ is considered and subdivided into excitation and radiation parts

$$F_i^{DI} = i\omega\rho \iint_S (\varphi_I + \varphi_D)\mathbf{h}_i\mathbf{n}\,\mathrm{d}S , \qquad F_i^{R} = \rho\,\omega^2 \sum_{j=1}^{N} \xi_j \iint_S \varphi_{Rj}\mathbf{h}_i\mathbf{n}\,\mathrm{d}S . \tag{9}$$

Thus, F_i^{DI} represents the modal pressure excitation. Now one can decompose (9) into the modal inertia force and damping force associated with acceleration and velocity, respectively

$$F_i^{a} = \mathrm{Re}(F_i^{R}) = \omega^2 \sum_{j=1}^{N} \xi_j A_{ij} , \qquad A_{ij} = \rho \,\mathrm{Re} \iint_S \varphi_{Rj}\,\mathbf{h}_i\mathbf{n}\,\mathrm{d}S , \tag{10}$$

$$F_i^{v} = \mathrm{Im}(F_i^{R}) = \omega \sum_{j=1}^{N} \xi_j B_{ij} , \qquad B_{ij} = \rho\,\omega\,\mathrm{Im} \iint_S \varphi_{Rj}\,\mathbf{h}_i\mathbf{n}\,\mathrm{d}S , \tag{11}$$

where A_{ij} and B_{ij} are elements of added mass and damping matrices, respectively.

Determination of added mass and damping for rigid body modes is a well-known procedure in ship hydrodynamics. Now the same procedure is extended to the calculation of these quantities for elastic modes. The hydrostatic part of the total pressure, $-\rho g z$ in (8), is considered within the hydrostatic model.

4 HYDROSTATIC MODEL

In spite of the fact that ship hydroelasticity has been a known issue for many years[7], there is still no unique solution for restoring stiffness[8, 9, 10, 11]. Here, its consistent formulation is presented in a condensed form.

The restoring stiffness consists of hydrostatic and gravity parts. Work of the hydrostatic pressure, which represents the generalized force, can be derived in the following form

$$F^h = -\rho g \iint_S \left[H_z + Z\left(\nabla \mathbf{H}\right)\right] \mathbf{H}\mathbf{n}\mathrm{d}S , \tag{12}$$

where ∇ is Hamilton differential operator, $\mathbf{H}$ is displacement vector, $\mathrm{d}S$ is differential of wetted surface, Z is its depth and $\mathbf{n}$ is unit normal vector. According to definition, the stiffness is relation between incremental force and displacement, so it is determined from the variational equation

$$\delta F^h = -\rho g \iint_S \left[H_z + Z\left(\nabla \mathbf{H}\right)\right] \delta \mathbf{H}\mathbf{n}\mathrm{d}S \cdot \tag{13}$$

Furthermore, the modal superposition method is used, and the variation is transmitted to modes, i.e. modal forces and displacements

$$\delta F^h = \sum_{j=1}^{N} \delta F_j^h, \quad \mathbf{H} = \sum_{j=1}^{N} \xi_j \mathbf{h}_j, \quad \delta\, \mathbf{H} = \sum_{j=1}^{N} \mathbf{h}_j \delta \xi_j \cdot \tag{14}$$

In that way, Eq. (13) is decomposed into the modal equations

$$\delta F_i^h = -\sum_{j=1}^{N} \left[\left(C_{ij}^p + C_{ij}^{nh}\right)\xi_j\right]\delta\xi_i , \tag{15}$$

where

$$C_{ij}^p = \rho g \iint_S \mathbf{h}_i h_z^j \mathbf{n}\mathrm{d}S , \qquad C_{ij}^{nh} = \rho g \iint_S Z \mathbf{h}_i \left(\nabla \mathbf{h}_j\right)\mathbf{n}\mathrm{d}S \tag{16}$$

are stiffness coefficients due to pressure, and normal vector and mode contributions, respectively.

Similarly to the pressure part, the generalized gravity force reads

$$F^m = -g \iiint_V \rho_s \left(\mathbf{H}\nabla\right) H_z \mathrm{d}V , \tag{17}$$

where ρ_s and V are structure density and volume, respectively. In order to obtain consistent variational equation, it is necessary to strictly follow the definition of stiffness and to vary displacement vector in (17) and not its derivatives

$$\delta F^m = -g \iiint_V \rho_s \left(\delta \mathbf{H}\nabla\right) H_z \mathrm{d}V \cdot \tag{18}$$

Application of the modal superposition method leads to the modal variational equation

$$\delta F_i^m = -\sum_{j=1}^{N} C_{ij}^m \xi_j \delta\xi_i , \tag{19}$$

where

$$C_{ij}^m = g \iiint_V \rho_s \left(\mathbf{h}_i \nabla\right) h_z^j \mathrm{d}V \tag{20}$$

are the gravity stiffness coefficients. Finally, the complete restoring stiffness coefficients are obtained by summing up its constitutive parts, $C_{ij} = C_{ij}^p + C_{ij}^{nh} + C_{ij}^m$.

5 HYDROELASTIC MODEL

After the structural, hydrostatic and hydrodynamic models have been determined, the hydroelastic model can be constituted. The governing matrix differential equation for coupled ship motions and vibrations is deduced

$$\left[\mathbf{k} + \mathbf{C} - i\omega\left(\mathbf{d} + \mathbf{B}(\omega)\right) - \omega^2\left(\mathbf{m} + \mathbf{A}(\omega)\right)\right]\boldsymbol{\xi} = \mathbf{F} , \tag{21}$$

where **k**, **d**, and **m** are structural stiffness, damping and mass matrices, respectively, **C** is restoring stiffness, $\mathbf{B}(\omega)$ is hydrodynamic damping, $\mathbf{A}(\omega)$ is added mass, $\boldsymbol{\xi}$ is modal amplitudes, **F** is wave excitation and ω is encounter frequency. All quantities, except ω and $\boldsymbol{\xi}$, are related to the dry modes. The solution of (21) gives the modal amplitudes ξ_i and displacement of any point of the structure obtained by retracking to (14).

6 COMPUTER PROGRAMS

Geometrical properties of ship hull cross-sections are determined by program STIFF[12], which is based on the advanced thin-walled girder theory. It calculates cross-section area, moments of inertia of cross-section, shear areas, torsional modulus, warping modulus and shear inertia modulus, for closed and opened cross-sections. The effective values of the above quantities can be also determined for the assumed sinusoidal modes.

For the hydroelastic analysis DYANA[13] program has been developed based on the advanced beam theory and finite element technique, taking shear, bending, pure torsion, shear torsion and warping of cross-section into account. The restoring stiffness is calculated according to Eqs. (16) and (20) for the deformed wetted surface, determined by spreading the beam deformation. The hydrodynamic part in DYANA is taken from program HYDROSTAR[14] and adopted for hydroelastic analysis.

7 HYDROELASTIC ANALYSIS OF CONTAINER SHIP

A large container ship of 11400 TEU, $L_{pp}\text{x}B\text{x}H = 348x45.6x29.74$ m, is considered. The equivalent torsional modulus due to influence of transverse bulkheads reads $I_t^* = 1.9I_t$. The reliability of the beam model is checked by correlating the natural frequencies and mode shapes with those of 3D FEM analysis performed by NASTRAN, Figs. 1 and 2.

Transfer functions of torsional moment and horizontal bending moment at the midship sections are shown in Figs. 3 and 4, respectively. They are compared to the rigid body ones determined by program HYDROSTAR. Very good agreement is obtained in the lower frequency domain, where the ship behaves as a rigid body. Discrepancies are very large at the resonances of the elastic modes.

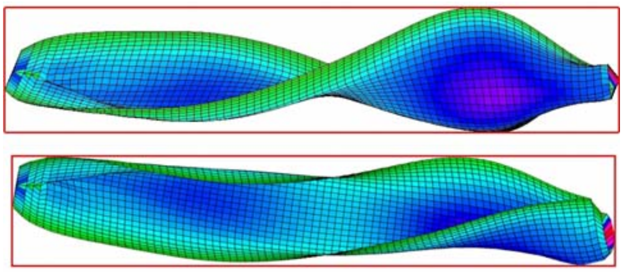

Figure 1: The first dominantly torsional mode, lateral and bird view, light weight, 1D model, ω_1=0.639 rad/s

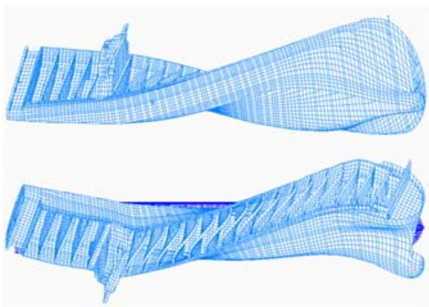

Figure 2: The first dominantly torsional mode, lateral and bird view, light weight, 3D model, ω_1=0.638 rad/s

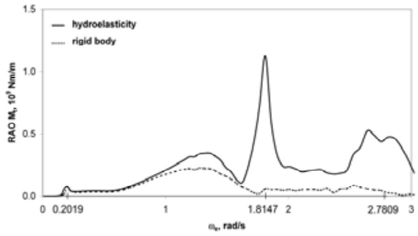

Figure 3: Transfer function of torsional moment, χ=120°, U=24.7 kn

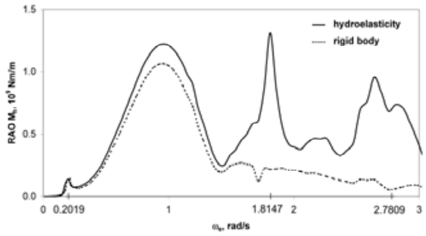

Figure 4: Transfer function of horizontal bending moment, χ=120°, U=24.7 kn

8 CONCLUSIONS

The illustrative numerical example of the 11400 TEU container ship shows that the developed mathematical model, utilizing the beam FEM model and 3D hydrodynamic model, is an efficient tool for application in ship hydroelastic analysis. The used sophisticated beam model, based on the advanced thin-walled girder theory with included shear influence on torsion and contribution of transverse bulkheads to torsional stiffness, is a reasonable choice in preliminary design stage. 3D FEM model of ship structure is preferable for final strength analysis and stress concentrations as a prerogative for fatigue estimation. Very good agreement between ship hydroelastic and rigid body responses in vicinity of zero encounter frequency, determined by the modal superposition method and direct integration, respectively, is obtained due to usage of the consistent restoring stiffness. Numerical procedure for ship hydroelastic analysis should be further evaluated by correlation analysis with model tests and full scale measurements, before its application for ship design.

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