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Wave induced hydroelastic behavior of the vertical circular cylinder with liquid filled tank at the top

Malenica Š.(1), **Choi Y.M.**(2), **Vladimir N.**(3), **Kwon S.H.**(4) **& Chen X.B.**(1)
(1) Bureau Veritas - Paris, France
(2) KRISO - Daejeon, Korea
(3) University of Zagreb - Croatia
(4) Pusan National University - Korea

Introduction

Global hydroelastic response of the ships carrying liquid cargo (LNG ships, tankers, dry cargo ships in ballast conditions...) can be influenced by the dynamics of liquid motions.The presence of the liquid cargo does not only change the structural natural frequencies but also introduce the additional resonant conditions due to the natural sloshing modes. In order to properly take into account the hydroelastic interaction in between two resonant systems (the floating body and the tanks) a fully coupled model should be used. The development of the fully consistent coupled hydroelastic model for general ship structure such as LNG carrier is quite complex and it is very usefull to have the reference data for validation. The purpose of this paper is to provide those data for the case of the simplified configuration which consists of the vertical circular cylinder with the liquid filled tank on its top. For this case it is possible to provide a highly accurated semi-analytical solution.

Mathematical model

The basic characteristics of this model were described in [4] and we recall them briefly below. Basic configuration is shown in Figure 1 and the modal appraoch is adopted for hydroelastic coupling [1]. The

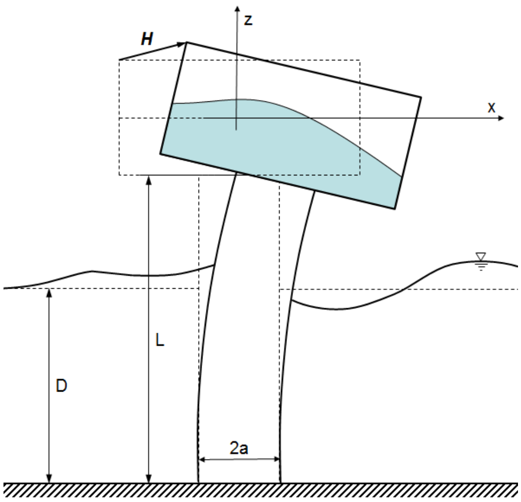

Figure 1: Basic configuration.

original six degrees of freedom dynamic system of the floating rigid body is extended with a certain number of elastic structural modes and we formally write for the displacement of one point on the body:

$$\boldsymbol{H}(\boldsymbol{x},\omega) = \sum_{j=1}^{N} \xi_j(\omega)\boldsymbol{h}^j(\boldsymbol{x}) \tag{1}$$

where N is the total number of modes (rigid + elastic), $\boldsymbol{x} = (x, y, z)$ describes the position of one point on the structure and $\boldsymbol{h}^j(\boldsymbol{x})$ is j^{th} modal displacement vector.

The seakeeping part of the analysis (i.e. without sloshing) leads to the following generalized dynamic equation:

$$\left\{-\omega^2([\ \mathbf{m}\ ]+[\ \mathbf{A}\ ])-i\omega[\ \mathbf{B}\ ]+([\ \mathbf{k}\ ]+[\ \mathbf{C}\ ])\right\}\{\boldsymbol{\xi}\}=\{\boldsymbol{F}^{DI}\} \tag{2}$$

where $[\ \mathbf{m}\ ]$ is the genuine dry body mass, $[\ \mathbf{k}\ ]$ is the structural stiffness, $[\ \mathbf{A}\ ]$ is the added mass matrix, $[\ \mathbf{B}\ ]$ is the damping matrix, $\{\xi\}$ is the modal amplitude vector and $\{\boldsymbol{F}^{DI}\}$ is the distributed pressure excitation (diffraction & incident). The dimension of the above linear system is $N = 6 + N_{flex}$ with N_{flex} denoting the number of flexible modes and the different hydrodynamic coefficients are calculated as described in [4].
The global body deformations will induce the tank deformations according to the different modal displacements and the liquid will move due to the velocities induced on the tank boundaries. The detailed analysis of the internal liquid problem [4] leads to the following boundary value problem (BVP) for the liquid motion inside the tank:

$$\left.\begin{array}{lr} \Delta\varphi^{T}_{Rj}=0 & \text{in the fluid} \\ -\nu\varphi^{T}_{Rj}+\dfrac{\partial\varphi^{T}_{Rj}}{\partial z}=\zeta_v^{Aj} & z=0 \\ \dfrac{\partial\varphi^{T}_{Rj}}{\partial n}=\boldsymbol{h}^j\boldsymbol{n} & \text{on } S_T \end{array}\right\} \tag{3}$$

where ζ_v^{Aj} denotes the vertical motion of the initial free surface for each elastic mode:

$$\zeta_v^{Aj}=\frac{\iint_{S_T}\boldsymbol{h}^j\boldsymbol{n}dS}{S_W} \tag{4}$$

The solution of the above BVP gives the linear sloshing potential from which the pressure can easily be calculated and integrated over the wet surface of the tank giving the tank added mass coefficients. This added mass matrix can be simply added to the dynamic motion equation (2).

Fully consistent solution and the rigid tank approximation

The critical technical point in the above sloshing analysis is the transfer of modal tank deformations from the structural mesh to hydrodynamic mesh in order to properly apply the body boundary condition. Once this acheived, the remaining analysis is rather straightforward and the numerical method presented in [3] can be applied directly. However, before considering this fully consistent model, here below we first introduce, what could be called, the rigid tank approximation.
The basic idea of this model is simple and consist in assuming that the tank is structurally rigid and will not be deformed by global body deformations but will move as a rigid body. In that case we can write for the motions of the representative point of the tank $\boldsymbol{x}_T$, (central bottom point in this case) and for the corresponding simplified modes, the following expression:

$$\boldsymbol{H}(\boldsymbol{x}_T,\omega)=\sum_{j=1}^{N}\xi_j(\omega)\boldsymbol{h}^j(\boldsymbol{x}_T)\quad,\quad \boldsymbol{h}^j(\boldsymbol{x})=\sum_{i=1}^{6}h_i^j(\boldsymbol{x}_T)\boldsymbol{h}^{Ri}(\boldsymbol{x}) \tag{5}$$

where it should be noted that the above expression includes both the translations and the rotations.
The local rigid body modes $\boldsymbol{h}^{Ri}(\boldsymbol{x})$ define the motion of the tank with respect to $\boldsymbol{x}_T$ so that the the sloshing potential can be written in the following form:

$$\varphi^{T}_{Rj}=\sum_{i=1}^{6}h_i^j(\boldsymbol{x}_T)\varphi^{R}_{Ri} \tag{6}$$

where φ^{R}_{Ri} represent the rigid body sloshing potential defined by (46) in [3].
Indeed, it is easy to show that, in this case, the vertical motion of the free surface inside the tank (4) becomes:

$$\zeta_v^{A1}=0\quad,\quad\zeta_v^{A2}=0\quad,\quad\zeta_v^{A3}=1\quad,\quad\zeta_v^{A4}=Y_A-Y_T\quad,\quad\zeta_v^{A5}=X_T-X_A\quad,\quad\zeta_v^{A6}=0 \tag{7}$$

With this in mind, the expression for the added masses becomes:

$$A^{T}_{ij} = \varrho \iint_{S_T} \varphi^{T}_{R_j} \boldsymbol{h}^i \boldsymbol{n} dS = \varrho \sum_{k=1}^{6} \sum_{l=1}^{6} h^j_k(\boldsymbol{x}_T) h^j_l(\boldsymbol{x}_T) \iint_{S_T} \varphi^{R}_{Rl} \boldsymbol{h}^{Rk} \boldsymbol{n} = \varrho \sum_{k=1}^{6} \sum_{l=1}^{6} h^j_k(\boldsymbol{x}_T) h^j_l(\boldsymbol{x}_T) A^{R}_{kl} \quad (8)$$

where A^{R}_{kl} is the rigid body added mass of the tank defined by (49) in [3].

Added mass matrix of the tank

In order to calculate the added mass of the sloshing motion of the tank, we use the approach described in [2]. The BVP for the evaluation of the linear sloshing potential is defined by (3) and, according to [2] the total potential is subdivided into 2 parts:

$$\varphi^{R}_{Rj} = \Omega_j + \phi_j \quad (9)$$

The first part of the potential Ω_j is the so called Stokes-Joukowski potential and it satisfies the Laplace equation in the fluid and rigid body boundary condition at all the boundaries i.e. including the free surface:

$$\frac{\partial \Omega_j}{\partial n} = \boldsymbol{h}^j \boldsymbol{n} \quad , \quad on \quad S_T + S_W \quad (10)$$

Note that this definition of the potential assumes that the tank is closed by the rigid lid on the free surface.
The remaining potential ϕ_j satisfy the homogeneous Neumann boundary condition on the tank boundaries S_T and the following condition on the free surface:

$$-\nu \phi_j + \frac{\partial \phi_j}{\partial z} = \zeta_v^{Aj} - h^j_z + \nu \Omega_j \quad (11)$$

Accordingly the total added mass is decomposed in two parts:

$$A_{ij} = A^{filled}_{ij} + A^{slosh}_{ij} \quad (12)$$

In the case of rectangular tank the solution for Ω is found by separation of variables and the solution for ϕ by natural modes expansion. The exact expressions for the potential and the corresponding added masses in the case of the rectangular tank are given in [2].

Few results

The characteristics of the vertical cylinder are those from [5] : cylinder radius is $a = 10m$, uniformly distributed mass along the length of the cylinder is the half of the displaced mass, a concentrated mass m_0 at the top of the cylinder (free surface level) is equal to the total displaced mass (liquid mass of the tank included), the stiffness of the cylinder is chosen such that the ratio EI/L^3 is equal to $0.41 m_0 s^{-2}$. Two sets of tank dimensions are chosen : (length$\times$ breadth $\times$ water level) = $(30 \times 30 \times 10)$ and $(23 \times 23 \times 10)$.
In Figure 2 we present the hydroelastic response of the cylinder (motion and slop of the top of the cylinder) with and without the presence of the tank. As expected we can observe the important influence of sloshing on the global system dynamics, especially close to the resonant tank modes. Indeed, when the wave frequency is equal to the first sloshing frequency of the tank the total response becomes zero. In addition, and as expected, the presence of the tank change the natural frequencies of the system and, in this particular cases, we can observe two separate peaks in the response contrary to the case of the "frozen liquid". Finally we can observe good agreement in between the semi-analytical and numerical results which confirms the correct implementation of the method in *Homer* software.

Conclusions and further work

We have presented here a method to take into account the influence of the sloshing on the global structural dynamics of the floating body. Both semi-analytical solution and the numerical one were built and they showed good agreement. The further work will consist in applying the method for general parctical cases

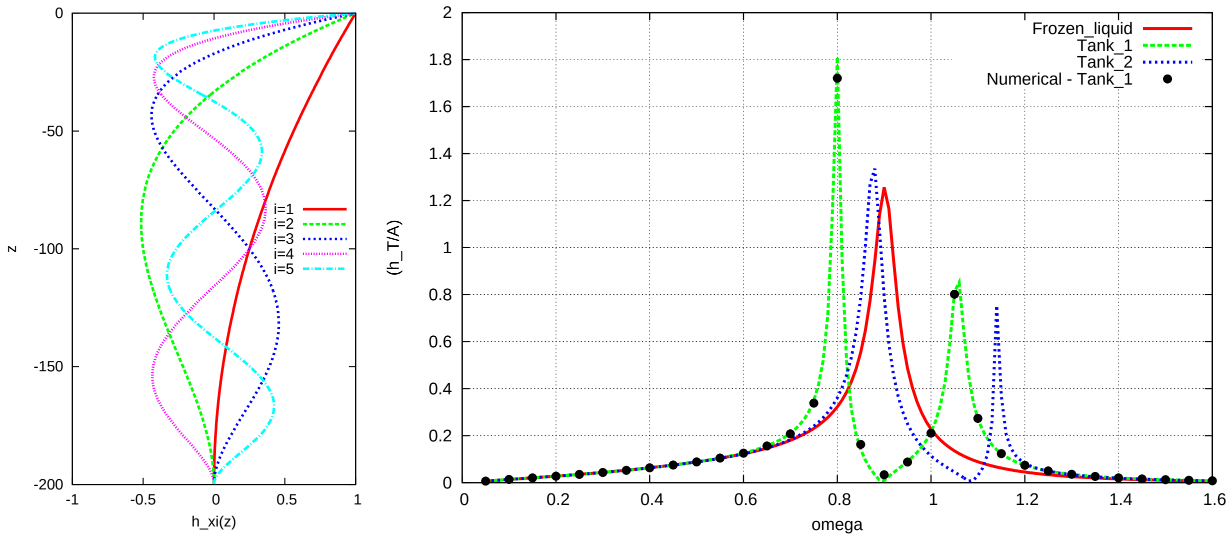

Figure 2: Deformation modes of the column without tank and linear RAO of the motion of the column top.

such as the LNG ships (Figure 3).
It is also important to mention one final particular point which was not discussed here and which concerns the hydrostatic restoring matrix. This point was briefly discusses in [4] but it was still not clear which expression should be used for the general case. Indeed, even without the liquid tanks, this remains a non trivial question which should be clarified in the future.

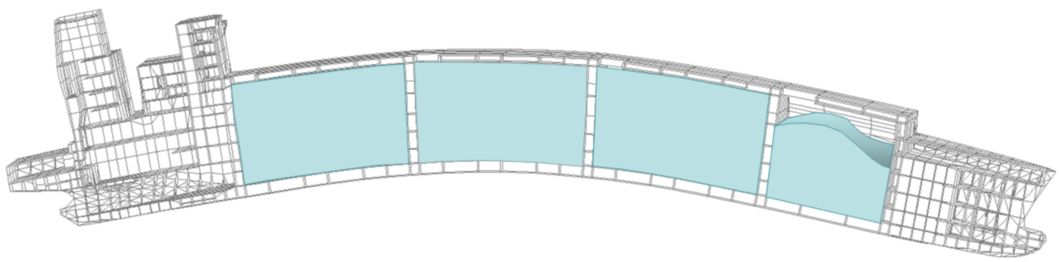

Figure 3: LNG ship with full and partially filled tanks.

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