6th IMSC
INTERNATIONAL MARITIME SCIENCE CONFERENCE
April 28nd-29rd, 2014 - Solin, Croatia

CALCULATION OF THE HYDRODYNAMIC LOADING ON A VERTICALLY SUBMERGED CYLINDER BY MEANS OF THE MORISON EQUATION

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ABSTRACT
Oil and natural gas are among the most important sources of energy. Price of these sources determines to a large extent all other branches of industry and global economy. The search for oil has resulted in the development of many technologies, primarily petroleum engineering and geosciences. Whereas land oil reserves are mainly exhausted, under oceans and seas there are vast deposits of oil and gas. This fact has influenced the development of marine engineering and extremely rapid progress of sea keeping. As a field of hydrodynamics, sea keeping theory researches the design and maintenance of offshore structures. Statistical analysis, wave models, force and energy calculations, structural analysis, etc. are various fields of research in sea keeping theory. In this paper the Morison equation, a semi-empirical equation for calculating the inline force on a body in oscillatory flow, has been used for calculating wave loads on submerged structures.

KEYWORDS
Fluids, Sea keeping, Waves, Morison equation.

INTRODUCTION

When designing a ship or a craft it is necessary to take into consideration the impact of waves exerting significant forces on the construction and affecting the construction's exploitation conditions. Analysis of wave loads is a field of research in sea keeping theory and, in a wider sense, fluid mechanics. Rough sea is considered as a random process as each wave has its own characteristics. Therefore it is hard to apply the laws of classical mechanics on a random process and it is necessary to simplify the problem, i.e. to transfer the different wave characteristics into a model.

LINEAR WAVE THEORY

It is necessary to reduce the complex wave action to a two-dimensional context in order to explain physical occurrences in the wave environment. The linear theory basically represents the solution of Laplace's equation, considering that the flow is defined, i.e. determined by boundary conditions, whether we deal with the vertical free surface or the bottom. In order to describe complex wave motion, it is necessary to define essential elements such as the potential of the flow velocity, pressure and forces on the wetted surface.

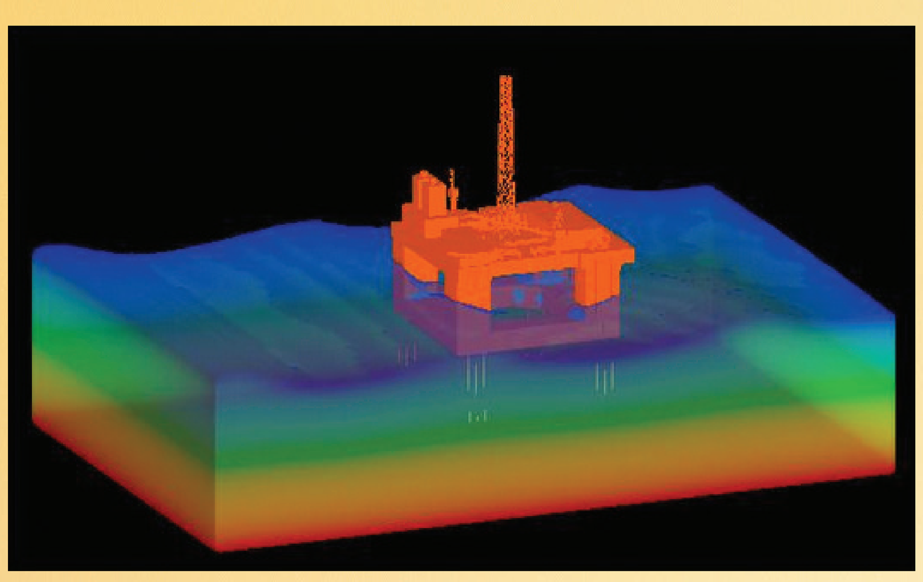

Figure 1. Simulation of oil rig loads resulting from wave action, *CFD Model*

Morison equation

The forces acting on a submerged, fixed vertical leg of an oil platform can be divided in two groups:

- Horizontal forces: drag force, inertia, and force of wave impact;
- Vertical forces: static and dynamic lift forces.

The forces having the most significant effect are the drag force and inertia, i.e. horizontal forces. The overall horizontal force exerted on the oil platform leg can be expressed as superposition of the drag force and inertia, assuming that the velocity and acceleration remain unchanged regarding the water depth Morison equation:

$$F = \int_{-h}^{n} dF = \int_{-h}^{n} \frac{1}{2} C_D \rho D u |u| dz + \int_{-h}^{n} \rho C_M \frac{\pi D^2}{4} \frac{Du}{Dt} dz$$

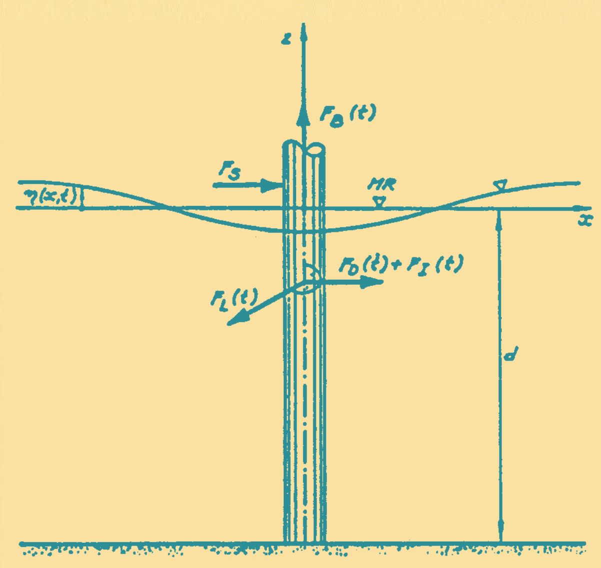

Figure 2. Main components of wave loading on the oil platform leg

Coefficients C_D and C_M depend on the leg's length. For the purpose of integration, they should be assumed as constant. This results in:

$$F = C_D DnE cos(kx_1 - \sigma t)|cos(kx_1 - \sigma t)| + C_M \pi DE \frac{D}{H} tanhkhsin(kx_1 - \sigma t)$$

CALCULATION OF THE LOAD ON AN OIL PLATFORM LEG

The following example shows the application of the hydromechanics postulates.

Analytical solution

A vertical, submerged cylinder is exposed to wave action. By applying Morison's equation it is possible to obtain an analytical solution for the values of forces and momentum that are dependent on time.

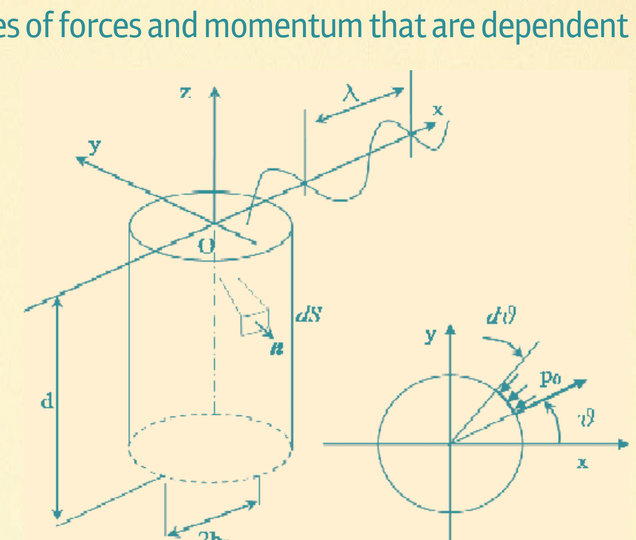

Figure 3. Loading on a vertical cylinder due to wave action [4]

Calculation has been performed for the leg having the following dimensions: $D_c = 2{,}5m$; $L = 28{,}0m$; $\lambda = 40{,}0$

The calculation has produced the maximum value of the force and momentum for $t = 1{,}4s$, $F_x = 47{,}24$ kN, $M_y = -1029{,}35$ kN

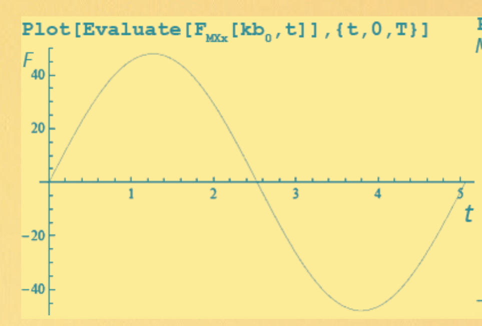

Figure 4. Diagram of force depending on time

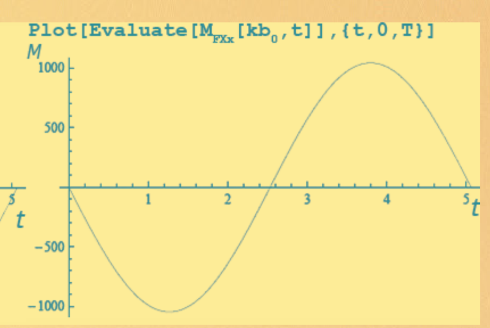

Figure 5. Diagram of momentum depending on time

Taking into consideration the coefficient value of the Froude – Krylov force in the diagram amounting to: $C_{MD} = 2$, the force achieves its final value: $F_x = 94{,}48$ kN.

Calculation performed in SESAM

Morison's model for calculating hydrodynamic load has been used to design the oil platform legs having the identical dimensions. The calculation has been carried out with the aid of the SESAM software package: F_x=116,09 kN, M_y=956,91 kNm, y=8,24 m

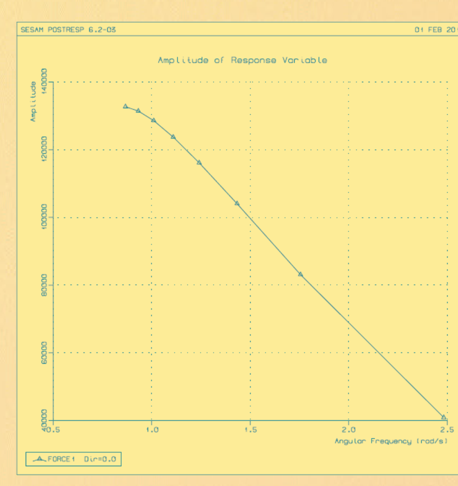

Figure 8. Diagram of the force towards the axis *x*

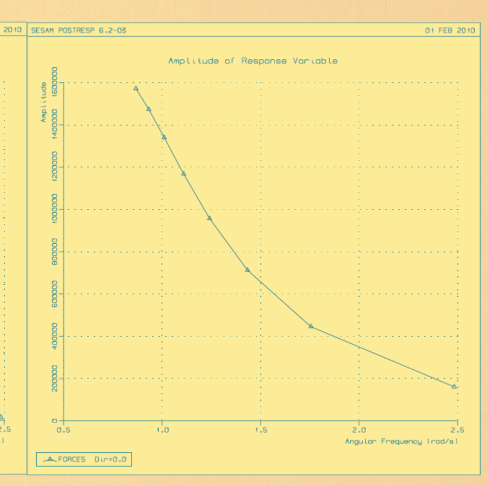

Figure 9. Diagram of the momentum around the axis *y*, *Nm* [4]

Table 6. Comparison of the results

	Analytical calculation	Morison's model	Measurement units
F_x	94,48	116,09	kN
M_y	-1029,35	956,91	kNm

Table 6 compares the results of the analytical calculation and the Morison's model in SESAM. For a selected wavelength (*λ=40,0 m*) the two methods produce values that vary by 18%. This variation is caused by a number of factors, primarily by the selection of coefficients. It can be noticed that the loading force value is smaller in the analytical calculation, which is logical, given the fact that the calculation can`t take into consideration all conditions of the hydrodynamic load. On the other hand, SESAM is able to accurately define the drag and added mass coefficients, i.e. the impact of inertia and drag forces, which results in a more precise, that is, a more realistic calculation. The analytical calculation assumes that the value of the added mass coefficient is 2. This value may not be optimal. It is likely that an added mass coefficient ranging from 0.7 to 0.8 would produce more realistic results.

CONCLUSION

This paper discusses the calculation of the hydrodynamic loading on a vertically submerged cylinder. The latter is assumed as a slender post-shaped structure that does not affect the wave field as the length of the cylinder is considerably less than the wavelength. Hydrodynamic loading has been calculated in an analytical way and by means of the Morison's 2D model in SESAM, and the obtained results have been compared. It can be concluded that the Morison model produces the results which indicate greater hydrodynamic loads due to inclusion of the drag and the added mass coefficients that can be accurately interpreted within the Morison's model. The simple example of a slender submerged structure has served for applying the Morison equation in the hydrodynamic loading calculation. The same approach can be used in designing and constructing much more complex offshore structures.