

## Fatigue growth models for multiple long cracks in plates under cyclic tension based on $\Delta K$ , $\Delta J$ -integral and $\Delta CTOD$ parameter

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**Abstract.** This paper presents the implementation of fatigue crack growth power law equations based on  $\Delta K$ ,  $\Delta J$ -integral and  $\Delta CTOD$  fracture mechanics parameters determined in an FE analysis, to plates with multiple site damage (MSD). Results of fatigue tests with constant amplitude tensile loading carried out on mild steel plate specimens damaged with a single central crack and with three collinear cracks are presented. A relatively larger plastic zone occurred in the crack tip region at higher fatigue crack growth rate (FCGR), from  $10^{-7}$  to  $10^{-6}$  m/cycle. The crack growth models based on the elastic-plastic fracture mechanics (EPFM) parameters describe better fatigue crack growth in this range as compared to the linear elastic models.

### Introduction

In thin-walled structures, fatigue cracks may initiate under a variety of loading and environmental conditions, at sites of stress concentration due to geometrical discontinuities. In ship deck structures very long cracks can occur due to the multisite damage (MSD), as the accident of Castor tanker showed [1]. The multisite damage can cause a disaster if allowed to progress [2]. From a damage tolerance design point of view, it is important to determine the fatigue crack growth in damaged structural parts.

A common approach to fatigue crack propagation analysis is to describe the crack growth rate by a differential equation, which is called a fatigue crack growth law or model. By integrating the differential equation one can obtain the crack length versus number of cycles,  $a-N$  curve, and predict the number of cycles required for the crack to grow from an initial to the final size. The well-known Paris law is based on the stress intensity factor (SIF) range,  $\Delta K$ , which represents the difference of the maximum and minimum  $K$  value,  $\Delta K = K_{\max} - K_{\min}$ , associated with the maximum and minimum applied nominal stress in a loading cycle,  $\sigma_{\min}$  and  $\sigma_{\max}$ , respectively [3]. Excessive plasticity during fatigue violates linear elastic fracture mechanics (LEFM) assumptions and the SIF  $K$  no longer characterizes the crack tip conditions. In such cases the elastic plastic fracture mechanics (EPFM) parameters  $J$ -integral and  $CTOD$  can be considered as a crack driving force. Dowling and Begley [4] used  $\Delta J$ -integral for fatigue crack growth modelling under large scale yielding conditions. Gasiak and Rozumek [5] presented a crack growth model based on  $\Delta J$ -integral and implemented it to various structural steel materials. Tanaka [6] demonstrated that  $CTOD$  can be a suitable parameter for fatigue crack growth modelling under elastic-plastic conditions.

In this paper calculated fracture mechanics parameters  $K$ ,  $J$ -integral and  $CTOD$  are presented, by using linear elastic (LE) and elastic plastic (EP) FE analysis for plate specimens with a single crack and an array of collinear cracks. The experiments carried out on mild steel specimens showed higher crack growth rates and relatively larger plastic zones in the vicinity of a crack tip for longer cracks. The intention was to implement fatigue crack growth models based on EPFM parameters  $J$ -integral and  $CTOD$  in modelling of higher crack growth rates observed in the experiment.

## Fatigue Crack Growth Models

In a typical fatigue crack growth rate curve,  $da/dN$  versus stress intensity factor range,  $\Delta K_I$ , one can distinguish three regions, commonly called region I, II and III. Region I is associated with an early fatigue crack development with FCGR typically of the order  $10^{-9}$  m/cycle or smaller. Region II represents a stable growth zone for long cracks where the data follow a linear relationship between  $\log da/dN$  and  $\log \Delta K$ , and the FCGR is typically in the range from  $10^{-9}$  up to  $10^{-6}$  m/cycle. Region III represents a zone of very high FCGR,  $da/dN > 10^{-6}$  m/cycle, associated with rapid and unstable crack growth.

Various fatigue crack growth prediction models have been developed to analyze propagation of long cracks. Paris used the  $\Delta K$  parameter to explain FCGR behaviour based on the LFM assumption. The Paris model does not take account of the SIF threshold,  $\Delta K_{th}$ , a  $\Delta K$  value below which crack growth practically does not occur. The  $\Delta K_{th}$  can be taken into account considering the effective part of the SIF range  $\Delta K_{eff}$ , as given by Eq. (1). This equation is also known as the Klesnil-Lukáš model [7]. In LFM the parameters  $K$  and  $J$ -integral are correlated. Assuming the plane stress conditions, the following relation exists between the two parameters:  $J=K^2/E$ , where  $E$  is Young's modulus. The modified Dowling and Begley model which takes into account the  $J$ -integral threshold,  $\Delta J_{th}$ , is represented by Eq. (2), where  $C_{db\ eff}$  and  $m_{db\ eff}$  are the material constants and  $\Delta J = J_{max} - J_{min}$ , is the  $J$ -integral range. The crack growth model based on  $\Delta CTOD$  parameter, which includes threshold  $\Delta CTOD_{th}$  values, is represented by Eq. (3).

$$\frac{da}{dN} = C_{p\_eff}(\Delta K^{m_{p\_eff}} - \Delta K_{th}^{m_{p\_eff}}) \quad (1)$$

$$\frac{da}{dN} = C_{db\_eff}(\Delta J^{m_{db\_eff}} - \Delta J_{th}^{m_{db\_eff}}) \quad (2)$$

$$\frac{da}{dN} = C_{ctod\_eff}(\Delta CTOD^{m_{ctod\_eff}} - \Delta CTOD_{th}^{m_{ctod\_eff}}) \quad (3)$$

The number of constant amplitude loading cycles due to which a crack grows from an initial crack length to a final crack length is determined by the integration of the Eq. (1-3).

## Experimental and Numerical Simulation Results

A plate specimen with a single central crack P1 and a plate specimen with three collinear cracks P3 were exposed to cyclic tension using a hydraulic fatigue testing machine. The specimen geometry is shown in Fig. 1.

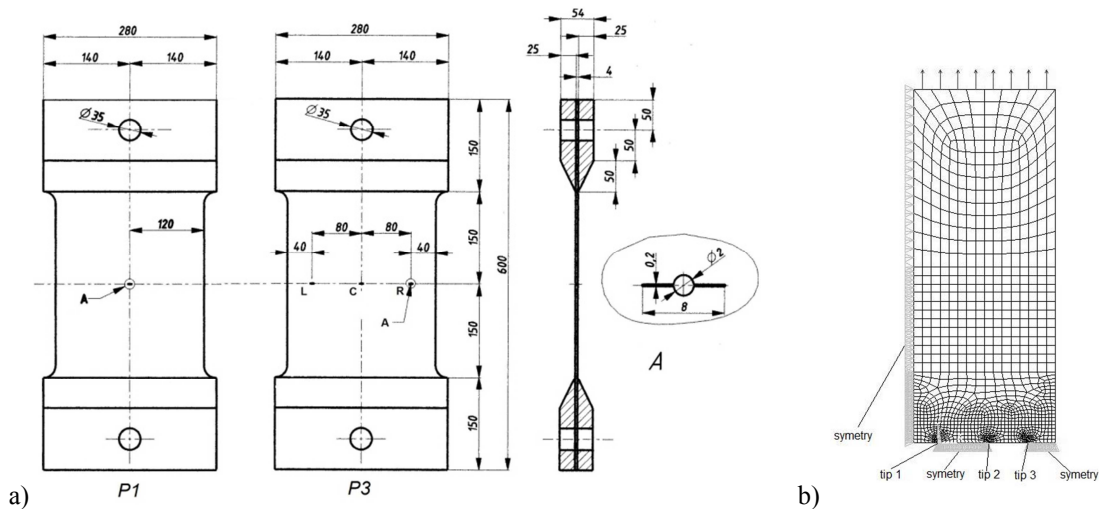


Figure 1 Fatigue test specimens P1 and P3: a) scantlings; b) FE mesh of P3 specimen.

The material used for the specimens is conventional mild steel for welded structures with the material properties specified as follows: ultimate strength is over 400MPa, yield strength is over 235MPa, Young's modulus is 206GPa, Poisson's ratio is 0.3, and the  $\Delta K_{Ih}=6.8\text{MPa}\sqrt{\text{m}}^{1/2}$ . The applied average stress range related to a cross-section in the intact area was  $\Delta\sigma_o = 80\text{ MPa}$ , with the loading frequency of 5 Hz, and the stress ratio,  $R = K_{\min}/K_{\max} = 0.025$ . The initial crack length was  $2a = 8\text{mm}$ . Crack length data presented here were measured by using an optical microscope.

Fracture mechanics parameters: the Mode I SIF values,  $K_I$ ,  $J$ -integral and  $CTOD$  were calculated by ANSYS [8] FEM software, where eight node quadratic isoparametric elements assuming plane stress conditions were used. Calculated  $\Delta K_I$  values for P1 specimens and for the crack tip 1 of P3 are given in Fig. 2.  $\Delta J$ -integral calculated by linear elastic and elastic plastic FEA for P1 and for the crack tip 1 of P3 is given in Fig. 3. In Fig. 4  $\Delta CTOD$  values for P1 and P3 specimens are given as obtained from the EP FEA. In Fig. 5 plastic zone size values are compared which were determined analytically under LEFM assumptions and numerically in the EP FEA. Elastic-plastic FEA results for  $J$ -integral values agree well with the values obtained from LE FEA for lower FCGR. A larger plastic zone in the crack tip region is generated at higher FCGR. The material constants for the Eq. (1-3) were determined from the rate diagrams given in Figs. 6-9.

Based on the fracture mechanics parameters given in Figs. 2-4, and in Eqs. (1-3), fatigue crack growth life curves have been simulated, as given in Fig. 10. The models based on the EPFM parameters describe better fatigue crack growth at higher FCGR in comparison with the LEFM models, since they provide a steeper a-N curve, as observed in the experiment.

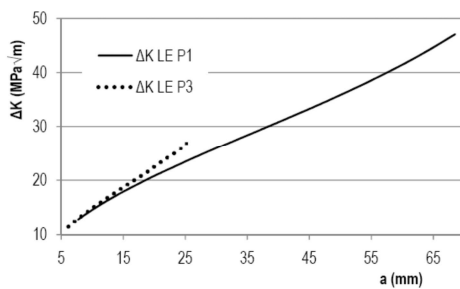


Figure 2  $\Delta K_I$  for P1 and the crack tip 1 of P3 specimen.

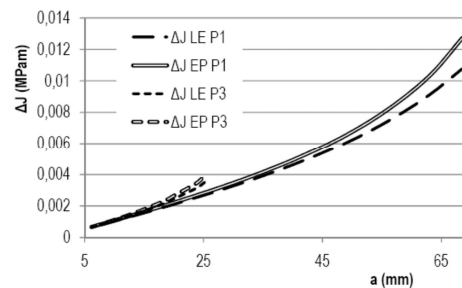


Figure 3  $\Delta J$  for P1 and the crack tip 1 of P3 specimen.

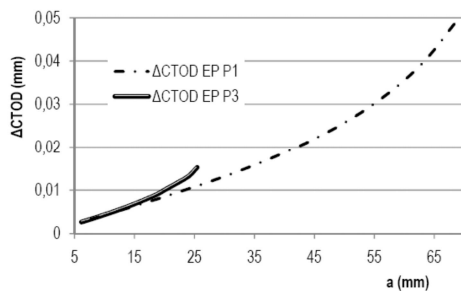


Figure 4  $\Delta CTOD$  for P1 and the crack tip 1 of P3.

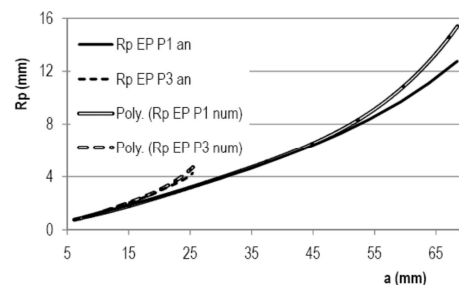


Figure 5 Plastic zone size  $R_p$ .

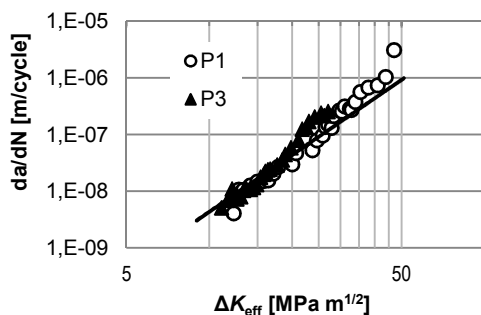


Figure 6 Rate diagram with respect to  $\Delta K_{\text{eff}}$ .

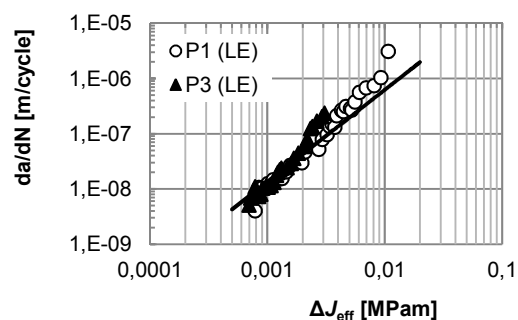


Figure 7 Rate diagram with respect to LE  $\Delta J_{\text{eff}}$ .

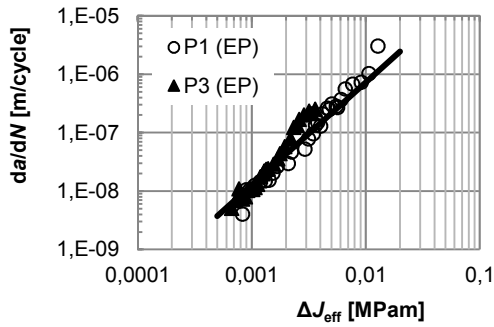


Figure 8 Rate diagram with respect to EP  $\Delta J_{\text{eff}}$ .

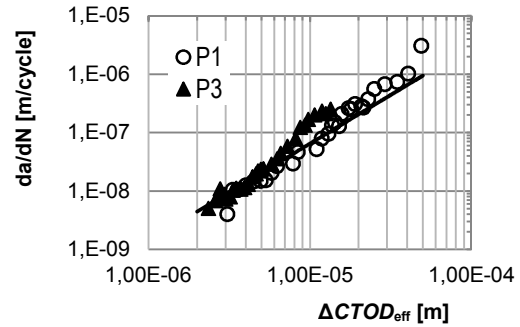


Figure 9 Rate diagram with respect to  $\Delta CTOD_{\text{eff}}$ .

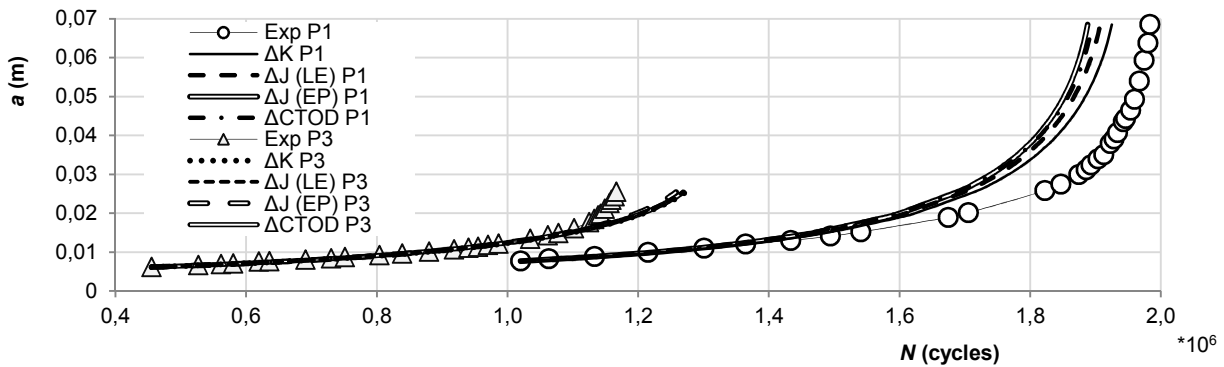


Figure 10 Fatigue life simulation results in comparison to experimental results for P1 and P3 specimens.

## Conclusion

Experiments on mild steel plate specimens damaged with a single central crack and with three collinear cracks showed a high fatigue crack growth rate prior to unstable crack growth and final collapse. Elastic-plastic FEA results for  $J$ -integral were close with linear elastic analyses results at lower FCGR. A relatively larger plastic zone is generated in the crack tip region at higher FCGR, from  $10^{-7}$  to  $10^{-6}$  m/cycle. The models based on the EPFM parameters describe better fatigue crack growth in this range as compared to the LEFM models.

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