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A SIMPLE HYBRID APPROACH ON PREDICTING THE VIBRATION TRANSMISSION OF RIB-STIFFENED PLATES

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Summary

Accurate predictions of vibration transmission of rib-stiffened plates can be quite problematic in that both long- and short-wavelength deformations are commonly generated within the same system model. In this case, it is widely acknowledged that hybrid methods which can combine both the deterministic and statistical modelling techniques are more appropriate than single deterministic or statistical methods. In this paper, a simple hybrid technique, i.e., the so-called "mode-based approach", is applied to predict the vibration transmission of a rib-stiffened plate model. The results are then compared with those of the well-established hybrid FE/SEA method and the exact modal analysis. It is found that the mode-based approach has a better performance than the hybrid FE/SEA method regarding both accuracy and efficiency, although the former is far less generic than the latter at the current stage.

Key words: *mid-frequency, mode-based approach, hybrid FE/SEA method, rib-stiffened plate, vibration transmission.*

1. Introduction

Accurate predictions of vibration transmission of rib-stiffened plates can be quite problematic in that long- and short-wavelength deformations are often generated simultaneously within the same system model. These problems are generally defined as the mid-frequency issue [1-2]. It is widely acknowledged [3] that, for the mid-frequency region, instead of a single deterministic method (e.g. the finite element method [4]) or statistical methods (e.g. the statistical energy analysis [5]), hybrid methods are more appropriate, e.g. as those described in [6-9].

Among the currently existing mid-frequency hybrid methods, one of the mostly developed and widely applied is the so-called "hybrid FE/SEA method" [10-12]. The main theoretical body of the method is so much well developed that it has been used to generate the mid-frequency module of the commercial software VA-One [13] for vibro-acoustic predictions of complex engineering systems. However, because of the way of describing the subsystem interface adopted in the hybrid modelling and the key theoretical assumption which underpins the theoretical development, the hybrid FE/SEA method is found to be time-consuming in the case of a large number of interface degrees of freedom (DOFs) between neighbouring subsystems [9], and also, significant discrepancy may be generated between the hybrid predictions and the measured data [14] if the SEA subsystems are of insufficient randomness. In these cases, more efficient and accurate hybrid techniques may be more desirable.

With this in mind, in the present paper another hybrid modelling technique, namely, “the mode-based approach”, which was developed in [15], is used to predict the vibration transmission of a simple rib-stiffened plate, as shown in Fig. 1, of which, the beam is assumed to be much stiffer than the plate. If an external harmonic force is applied at a particular point of the beam, the system will exhibit significant mid-frequency vibration behaviour over a large frequency range. The performance of the mode-based approach can then be valued by comparing the results with those from the hybrid FE/SEA method and the exact modal analysis [6, 15].

The paper is organized as follows. First, the main principles of the two hybrid modelling techniques are briefly explained in Section 2, with their major theoretical differences being discussed. Then in Section 3, a simple rib-stiffened plate model with a line-connection is set up, and the power transmission of the system model is calculated by both hybrid techniques and compared to the results from the exact modal analysis. Finally, conclusions are deduced in Section 4.

2. Main theoretical principles of the two hybrid modelling techniques

In this section, only the main theoretical principles of and the major differences between the two hybrid models are briefly recapitulated. Detailed theoretical derivations and the main results are not given for the brevity of the paper, as they can be found very conveniently in Refs. [10] and [15]. Nevertheless, the main theoretical results of the two methods corresponding to a simple beam-plate coupling model are given so as to enable reading without difficulty.

2.1 Hybrid FE/SEA method

In the hybrid FE/SEA method [10], long-wavelength subsystems are described by deterministic FEA models, while the short-wavelength ones are described by the statistical SEA models. Each individual SEA subsystem is further divided into a “direct field” and a “reverberant field” in that the former accounts for the in-going waves without reflections while the latter for the out-going waves due to the reflections from its boundaries. A diffuse field reciprocity relation is then established between the reverberant force loading on the deterministic boundaries of the SEA subsystem and the energy response of the subsystem under the assumption of the non-deterministic boundaries of the subsystem being sufficiently random [11]. This thus allows a statistical relation to be formed between the dynamic response of the FE subsystems and the energy response of the SEA subsystems, upon which the whole set of hybrid FE/SEA equations are built-up [12].

It is seen that the key assumption which underpins the hybrid FE/SEA theory is that each SEA subsystem needs to be sufficiently random so that the diffuse field reciprocity relation can be satisfied. However, there are no explicit measures on determining the randomness level of a particular subsystem. This difficulty can sometimes make the validity range of the hybrid FE/SEA predictions, to a certain degree, ambiguous.

It is also worth noting that all the interfaces between subsystems (including those between SEA subsystems) are defined as parts of the deterministic models in [10]. When the subsystem interfaces contain a large number of degrees of freedoms (DOFs), e.g. for continuous connections between the FE and SEA subsystems, the matrix size of the deterministic model can still be extremely high. In this case, high computational cost also seems unavoidable [9, 16].

2.2 Mode-based method

The mode-based approach [15] describes subsystem components in a very similar way to the hybrid FE/SEA theory in that the long-wavelength components are described deterministically while the short-wavelength components are approximated statistically. But in the mode-based approach, instead of partitioning each SEA subsystem into a direct field and a reverberant field, each SEA subsystem is simply approximated as a set of sinusoidal waves with randomly varied phases. Because previous research has shown that the higher ordered mode shapes functions tend to behave as simple standing waves regardless of the exact boundary conditions and shapes of the structure [17-19], it provides a solid theoretical foundation to simulate the mode shapes of the short-wavelength receiver as sinusoidal waves.

Another significant difference between the two hybrid techniques is that in the mode-based approach the interfaces between the long- and short-wavelength subsystems are treated approximately by sets of generalized interface modes [15, 20] rather than as a part of the deterministic model. Previous research reveals that, by choosing the interface mode set properly, the number of the interface modes required for a continuous coupling interface can be much less than the number of interface DOFs [20]. Therefore, by projecting the physical interface DOFs into the space of generalized interface modes, the size of the generated hybrid model can be dramatically reduced [15, 21].

Despite of the two obvious advantages of the mode-based approach over the hybrid FE/SEA method, the former approach was relatively much less generalized than the latter.

2.3 Theoretical application to a simple rib-stiffened plate model

In order to demonstrate the application of the above presented two hybrid modelling techniques, a simple rib-stiffened plate structure, as shown in Fig. 1, is set up in the first instance. A stiff beam (rib) is wholly attached to a flexible plate. When external excitations are applied directly onto the beam, the beam-plate structure will possess typical mid-frequency vibration behaviour for a quite large frequency range. In this case, either the hybrid FE/SEA theory or the mode-based approach is appropriate to be employed to predict the power transmitted within the system. The relevant equations are given below.

2.3.1 Power transmission from the beam to the plate based on the hybrid FE/SEA theory

Under the condition of the plate subsystem being sufficiently random, a statistical power balance equation can be set up for the plate in the sense of ensemble- and time-averages [10], as

$$\omega\left(\eta_p+\eta_{b,p}\right)E_p=P_{in,p}^{ext}. \tag{1}$$

In the above equation, E_p is the statistical energy response of the plate, and η_p and $\eta_{b,p}$ are the damping loss factor of the plate and the power transfer coefficient between the plate and the beam, respectively. On the right side of the equation, $P_{in,p}^{ext}$ represents the power injected into the plate by the external force acting on the beam. These parameters can be calculated as

$$\eta_{b,p}=\left(\frac{2}{\pi\omega n_p}\right)\sum_{rs}\operatorname{Im}\left\{\mathbf{D}_{b,rs}\right\}\left(\mathbf{D}_{tot}^{-1}\operatorname{Im}\left\{\mathbf{D}_{dir}^{(p)}\right\}\mathbf{D}_{tot}^{-1*T}\right)_{rs}, \tag{2}$$

$$P_{in,p}^{ext} = \frac{\omega}{2}\sum_{rs}\operatorname{Im}\left\{\mathbf{D}_{dir,rs}^{(p)}\right\}\left(\mathbf{D}_{tot}^{-1}\mathbf{S}_{ff}\mathbf{D}_{tot}^{-1*T}\right)_{rs}, \tag{3}$$

$$\mathbf{D}_{tot} = \mathbf{D}_b + \mathbf{D}_{dir}^{(p)}, \tag{4}$$

where, both r and s are integers, representing the degrees of freedom (DOFs) of the beam, and n_p and $\mathbf{D}_{dir}^{(p)}$ are the modal density and direct field dynamic stiffness matrix of the plate, respectively, while $\mathbf{D}_d$ is the dynamic stiffness matrix of the beam when it is uncoupled from the plate. The superscripts $*$ and T represent complex conjugate and matrix transformation, respectively. $\mathbf{S}_{ff}$ in Eq. (3) is the ensemble-averaged cross-spectral matrix of the external excitation force vector $\mathbf{f}$, i.e.,

$$\mathbf{S}_{ff} \equiv \mathrm{E}\left[\mathbf{ff}^{*T}\right], \tag{5}$$

where, $\mathrm{E}[\cdot]$ represents ensemble average.

Substituting Eqs. (4)-(5) into Eqs. (2)-(3), then into Eq. (1), the statistical energy response of the plate E_p can be determined.

Then, by the statistical energy theory described in [5], the transmitted power from the beam to the plate can be calculated as

$$P_{tr,p} \approx \omega\eta_p E_p = \frac{P_{in,p}^{ext}}{\left(1+\eta_{b,p}/\eta_p\right)} \tag{6}$$

It can be seen clearly from Eq. (4) that the hybrid FE/SEA predicting procedure can still be quite time-consuming if the beam-plate interface contains a large number of DOFs due to the corresponding large matrix size of $\mathbf{D}_{tot}$.

2.3.2 Power transmission from the beam to the plate based on the mode-based approach

In the mode-based approach [15], if the dynamics of the interface between the source (beam) and the receiver (plate) are described by a set of generalized interface modes $\mathrm{X}_{I,k}$ set of interface modes along the interface $\mathbf{x_I}$, the corresponding generalized interface force-coordinates and displacement coordinates can then both be written in the matrix forms as

$$\mathbf{f_{I,k}} = \left[\mathbf{A_b} + \mathbf{A_p}\right]^{-1} \boldsymbol{\alpha_b}^T \mathbf{Y_{b,n}} \mathbf{f_{e,n}}, \tag{7}$$

$$\mathbf{w_{I,k}} = \mathbf{A_p}\left[\mathbf{A_b} + \mathbf{A_p}\right]^{-1} \boldsymbol{\alpha_b}^T \mathbf{Y_{b,n}} \mathbf{f_{e,n}}. \tag{8}$$

In the above equations, $\boldsymbol{\alpha_b}$ and $\boldsymbol{\alpha_p}$ are both matrices, whose elements can be determined as

$$\alpha_{b,nk} = \int_{V_b^I} \Phi_{b,n}\left(\mathbf{x_b^I}\right)\mathrm{X}_{I,k}\left(\mathbf{T}_b^T\mathbf{x_b^I}\right)d\mathbf{x_b^I}, \tag{9}$$

$$\alpha_{p,mk} = \int_{V_p^I} \Psi_{p,m}\left(\mathbf{x_p^I}\right)\mathrm{X}_{I,k}\left(\mathbf{T_p}^T\mathbf{x_p^I}\right)d\mathbf{x_p^I}, \tag{10}$$

where $\Phi_{b,n}$ and $\Psi_{p,m}$ are the mode shape functions of the beam and the plate at the local interface locations $\mathbf{x}_\mathbf{b}^\mathbf{I}$ and $\mathbf{x}_\mathbf{p}^\mathbf{I}$, respectively, and $\mathbf{T_b}$ and $\mathbf{T_p}$ represent transformation matrices which relate the local coordinates of $\mathbf{x}_\mathbf{b}^\mathbf{I}$, $\mathbf{x}_\mathbf{p}^\mathbf{I}$ and $\mathbf{x_I}$ with $\mathbf{x}_\mathbf{b}^\mathbf{I} = \mathbf{T_b x_I}$ and $\mathbf{x}_\mathbf{p}^\mathbf{I} = \mathbf{T_p x_I}$. Furthermore, matrices $\mathbf{A_b}$ and $\mathbf{A_p}$ are given by

$$\mathbf{A_b} = \boldsymbol{\alpha}_\mathbf{b}{}^T \mathbf{Y_{b,n}} \boldsymbol{\alpha}_\mathbf{b}, \tag{11}$$

$$\mathbf{A_p} = \boldsymbol{\alpha}_\mathbf{p}{}^T \mathbf{Y_{p,m}} \boldsymbol{\alpha}_\mathbf{p}. \tag{12}$$

In the above two equations, $\mathbf{Y_{b,n}}$ and $\mathbf{Y_{p,m}}$ are both diagonal matrices whose diagonal elements are determined by the corresponding modal receptances of the uncoupled subsystems $Y_{b,n}$ and $Y_{p,m}$, respectively, i.e.,

$$Y_{b,n} = \frac{1}{m_{b,n}\left[\omega_{b,n}^2\left(1+j\eta_{b,n}\right)-\omega^2\right]}, \tag{13}$$

$$Y_{p,m} = \frac{1}{m_{p,m}\left[\omega_{p,m}^2\left(1+j\eta_{p,m}\right)-\omega^2\right]}, \tag{14}$$

where, $\omega_{b,n}$ and $\omega_{p,m}$ are the nth and the mth natural frequencies of the uncoupled source and receiver subsystems, respectively, while $m_{b,n}$, $\eta_{b,n}$ and $m_{p,m}$, $\eta_{p,m}$ are the corresponding modal mass and modal damping loss factors, respectively. Finally, $\mathbf{f_{e,n}}$ in Eqs. (7)-(8) is a column vector composed of the modal forces of the beam. The nth modal force of the beam can be determined as

$$f_{e,n} = \int_{V_b^e} F_e\left(\mathbf{x}_\mathbf{b}^\mathbf{e}\right)\Phi_{b,n}\left(\mathbf{x}_\mathbf{b}^\mathbf{e}\right)d\mathbf{x}_\mathbf{b}^\mathbf{e}. \tag{15}$$

For instance, if a unit harmonic force acting at point ξ of the beam, the corresponding modal force of the beam, by Eq. (15), can be determined as $f_{e,n} = \Phi_{b,n}\left(\xi\right)$, where $\Phi_{b,n}$ is the nth mode shape function of the beam.

The time-averaged power transmitted from the beam (source) to the plate (receiver) can thus be expressed as

$$P_{tr,p} = \frac{1}{2}\mathrm{Re}\left\{ j\omega\sum_k w_{I,k} f_{I,k}^* \right\}. \tag{16}$$

It can be seen clearly from Eqs. (7)-(8) that, by introducing a set of generalized interface modes, the mode-based approach can reduce the large interface DOFs into generalized interface coordinates. Meanwhile, by statistically approximating the receiver as a simple standing wave system, the generated modes are to be easily estimated based on the wavelength within a subsystem, while the natural frequencies are to be estimated from the free wavenumber within the subsystem.

3. Numerical investigations

In Fig. 1, a stiff beam is wholly attached to a flexible plate with a unit harmonic force acting at the point ξ of the beam. For simplicity, the beam is assumed to have both ends free, while the plate is assumed to have four edges simply supported. Moreover, the neutral axis of the beam is assumed to be lying in the mid-plane of the plate so that the dynamic effects arising from the longitudinal coupling of the system can be taken as relatively very small compared to those of the flexural coupling. Consequently, the system vibration can be simply treated as involving only flexural wave motions.

Firstly, the time-averaged power transmission from the beam to the plate is predicted by the following methods: (1) the exact modal analysis, (2) the mode-based approach, and (3) the hybrid FE/SEA method. Comparisons are then made for the three sets of results in terms of both the accuracy and the computational cost.

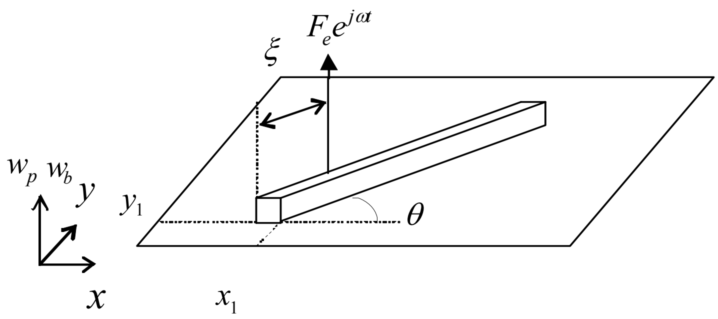

Fig. 1 A beam-plate model with line-connections

For method (1), the modal properties of both the (free-free) beam and the (simply supported thin rectangular) plate can be determined by their analytical expressions [18]. This then allows the exact expressions for the frequency response functions (FRFs) of the beam and the plate to be formed [19]. By means of the displacement continuity and force equilibrium conditions along the interface, the whole response of the system model can then be predicted in an exact manner. For methods (2) and (3), the relevant equations used in the calculations have been provided in Subsections 2.3.1 and 2.3.2, respectively. Moreover, during the calculations of Methods (1) and (3), the line interface between the beam and the plate is simulated by uniformly distributed points which are uniformly spaced at no more than a quarter wavelength (of the plate) apart along the coupling line. However, in Method (2), the line interface is represented by using the first 20 bending modes of the free-free beam to form the space of the generalized interface modes, while the mode shape functions of the plate are simulated as double sinusoidal functions but with arbitrary random phases.

Here it should be noted that, because the hybrid FE/SEA equations were derived in the sense of ensemble averages [10], frequency averages need to be taken from the results of the mode-based approach and those of the exact method in order to compare the three sets of results properly. The bandwidth employed here for the frequency average is 20Hz.

3.1 Model description

In Fig. 1, for simplicity, the beam is assumed to have both ends free, while the plate is assumed to have four edges simply supported. Also, the calculation procedure only involves the bending motions of the system.

The system material properties are given in Table 1 and the dimensional sizes of the beam and the plate as well as their interface locations are listed in Table 2.

The frequency range of interest is up to 1000 Hz, which composes about 6 bending modes of the beam, and 285 and 115 bending modes for the plates with the plate thickness of 2 mm and 5 mm, respectively. Obviously, all the three coupling cases involved possess typical mid-frequency vibration behaviour for a large part of the frequency range of interest.

3.2 Numerical results

The statistical power transmission from the beam to the plate calculated by the two hybrid techniques and the exact analytical theory are compared in Figs. 2-3, corresponding to the two coupling cases of $\lambda_b/\lambda_p = 4{,}37$ and $\lambda_b/\lambda_p = 2{,}76$, respectively. In the figures, the real lines are given by the exact modal analysis, and the dotted lines correspond to the results of the mode-based approach, while the dashed lines are calculated by the hybrid FE/SEA method.

It is seen that most of the significant peaks and troughs of the accurate response (real line) of the system have been well-captured by the hybrid solution (the dashed line) in Fig. 2, but in Fig. 3, the agreement between the real and the dash lines can be observed as generally poor. It is thus shown clearly that the predicting accuracy of the hybrid FE/SEA method is much better in Fig. 2 than in Fig. 3. This is because the plate in Fig. 2 is much more flexible than that used in Fig. 3, and consequently, the plate in Fig. 2 is more able to meet the "sufficient randomness" condition than the one in Fig. 3. Therefore, one can deduce that, to apply the hybrid FE/SEA theory successfully to rib-stiffened plate systems, a plate component with sufficient randomness is essential.

However, comparing the predictions of the mode-based approach (dotted line) and the exact solutions (real lines) in both Fig. 2 and Fig. 3, it can be seen that the mode-based approach can make a fairly good prediction of the transmitted power even for the relatively lower frequency range. For instance, a good agreement between the dotted and the real lines can be observed in Fig. 3 from a frequency region lower than 200Hz, where the hybrid FE/SEA solution nearly loses its validity completely. It is thus clearly revealed that the mode-based approach has a much better accuracy than the hybrid FE/SEA method, especially for the frequency region where the plate randomness level is generally low. This can be reasonably explained in the way that the "simple standing-wave" approximation used in the mode-based approach for the flexible plate subsystem is relatively much easier to be sustained than the "sufficient randomness" assumption used in the hybrid FE/SEA theory. As a result, the application condition of the mode-based approach can be relatively much more relaxed than that of the hybrid FE/SEA method.

Moreover, when calculating the data in Fig. 2, the computational cost of the mode-based approach (0,4721 min) was found only about 50% of that of the hybrid FE/SEA method (0,9534 min), and 8% of the exact method (5,6167 mins). But this time-saving advantage of the mode-based approach becomes weaker when the plate gets stiffer. When calculating the data in Fig. 3, for instance, the time taken by the mode-based approach, the hybrid FE/SEA method and the exact solution are 0,1529 min, 0,2813 min and 1,2068 mins, respectively. In this case, the computational cost of the mode-based approach becomes 55% and 13% of those of the hybrid FE/SEA method and the exact technique, respectively. One can thus deduce that the mode-based approach becomes more accurate and more efficient as the plate wavelengths become shorter.

Table 1 Material properties of the beam/plate model in Fig. 1

Young's modulus, GPa	Poisson's ratio	Loss factor	Density, kg m^{-3}
210	0,3	0,01	7850

Table 2 Dimensions of the beam-plate model and the excitation location on the beam

Structure	Beam	Plates
Dimensions, m	Length $L_b = 2{,}0$, Width $t_b = 0{,}030$, Height $h_b = 0{,}040$.	Length: $L_x = 2{,}0$, Width: $L_y = 0{,}9$, Thickness: $h_p = 0{,}002$, $h_p = 0{,}005$ $x_1 = 0{,}02$, $y_1 = 0{,}30$, $\theta = 10°$.
Excitation, m	$\xi = 0{,}73$	
Wavenumber ratio	$\lambda_b / \lambda_p = 4{,}37$, $\lambda_b / \lambda_p = 2{,}76$	

3.3 Discussion of the results

The good agreements in Figs. 2-3 between the mode-based approach and the exact modal analysis illustrate clearly that the mode-based approach can provide very efficient predictions of the power transmission of rib-stiffened plate systems, especially when the modal density of the plate is not high enough so that a sufficient randomness condition cannot be well-satisfied. Comparing with the hybrid FE/SEA solutions in Figs. 2-3, the mode-based approach also shows a much better performance not only regarding accuracy but also efficiency.

Although the comparisons are only made for the simplest beam-plate models, the results show that the mode-based approach has a great potential to be a good supplement to the existing hybrid modelling techniques.

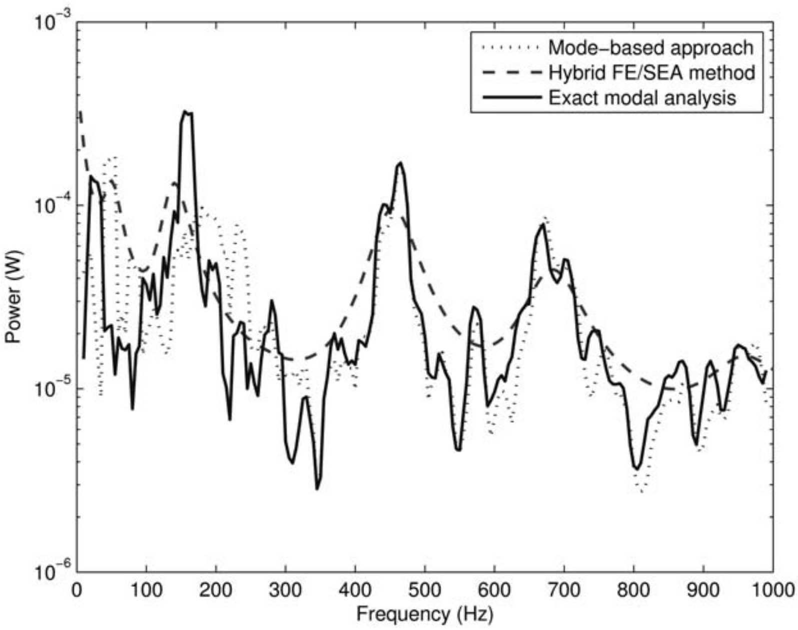

Fig. 2 Transmitted power from the beam to the plate when the plate thickness is 2 mm

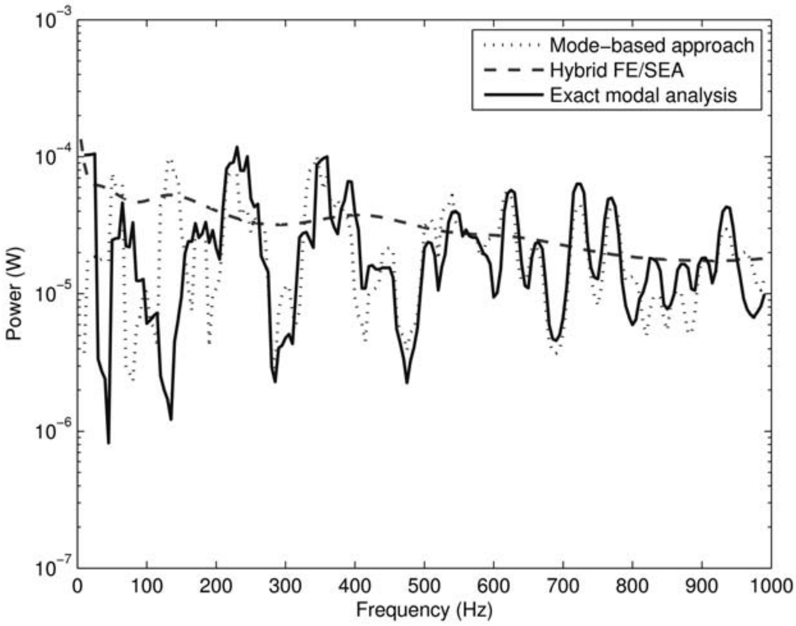

Fig. 3 Transmitted power from the beam to the plate when the plate thickness is 5 mm

4. Concluding remarks

In this paper, a simple hybrid technique, the so-called "mode-based approach", is applied to predict the vibration transmission of a rib-stiffened plate model. The results are then compared with those of the well-established hybrid FE/SEA method and the exact modal analysis. It is found that the mode-based approach has a much better performance than the hybrid FE/SEA method regarding both accuracy and efficiency.

Having said this, however, the mode-based approach is far less generic than the hybrid FE/SEA method as the latter has been widely applied to many complex built-up systems in practical engineering [13]. Despite this fact, the derivation procedure in [15] has shown that the mode-based approach can be extended to generic complex built-up systems with significant local dynamic mismatches in a quite straightforward way. Further research on generalizing the mode-based approach is underway.

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