# PARKING COMPLEX PROFIT ESTIMATION

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ABSTRACT

Salary from the car parking business is mostly known only for the authorities who are charging the money. The mathematics model for external estimating the profit from parking payment on a registered parking place is presented and application is demonstrated. Considering the usually camera partly monitoring, authors are giving the method for general salary calculation based on regression analysis.

KEY WORDS: profit, integral, trapezium formula, correlation, regression, profit control.

## INTRODUCTION

The whole income from the parking place is well known only for a subject who is taking it. Failure of this concept is in possible manipulating by origin data, especially if one public firm has many parking complexes in a city. This problem arises when observation time is getting longer.

Parking place occupancy monitoring is possible by video cameras. At the base of recording video snapshots it is possible to calculate, as precise as is required, the exact profit taken from the recorded parking place. If the cameras are able to cover only the part of the parking complex, it is possible to make advanced measuring and estimate the salary by regression analysis.

We use manuals from University of Zagreb. In all literature we considered there is no any similar problem investigating. So this article overgrows from an exact problem we considered.

## MATHEMATICAL MODEL

The exact cumulate profit *C* from the parking place complex is depending on the cumulate time that the every car is spending on its parking place:

$$C = c_1 \cdot \sum_i t_i$$

where *i* is a parking car, $t_i$ is the hours that the *i*-th car is spending in its parking place and $c_1$ is the price of one parking hour. When we create the mathematical model, we suppose that every driver is paying his parking. It is because we have only the record of parking with places that are occupied by cars and empty parking places.

Let *a(t)* be the number of parking cars in a moment *t* which could be taken from the snapshoot recorded during the parking payment period. If there exist a formula *a(t)* depending on a variable *t,* then

$$C = c_1 \int_{t_1}^{t_2} a(t)dt \qquad (1)$$

with $[t_1,t_2]$ as observed snapshoot interval, usually the daily parking charging time. Under the very special assumption that $a(t)$ is constant during the period from $t_1$ to $t_2$, we get the calculus

$$C = c_1 \int_{t_1}^{t_2} a(t)dt = c_1 \int_{t_1}^{t_2} adt = c_1 a \int_{t_1}^{t_2} dt = c_1 at \Big|_{t_1}^{t_2} = ac_1 (t_2 - t_1). \qquad (2)$$

Notice that value $C$ is an product of an hour parking price $c_1$ and the value of area shown on the Figure 1:

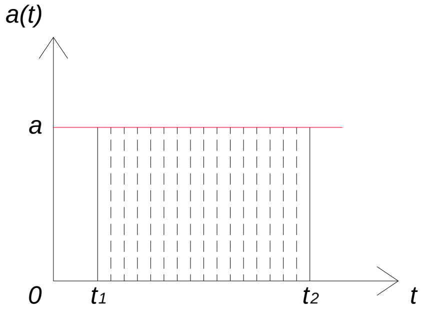


*Figure 1. Area in a constant car number case*

In general, the number of cars in every moment is variable, but $a(t)$ continuously depends on $t$. So the value given in (1) is again the product of an hour parking price and the value of area shown on the Figure 2:

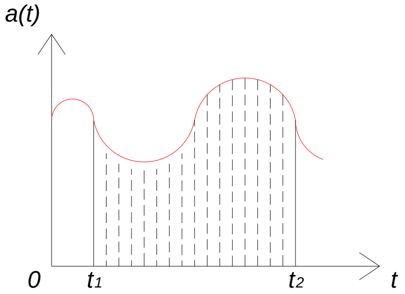


*Figure 2. Area in variable continuous car number case*

Parking cars can be counted in discrete moments $t_1, t_2, t_3, \ldots, t_n$. So there are discrete values of $a(t)$: $a_1, a_2, a_3, \ldots, a_n$. The values between them are approximate by polygonal line as is shown on the Figure 3:

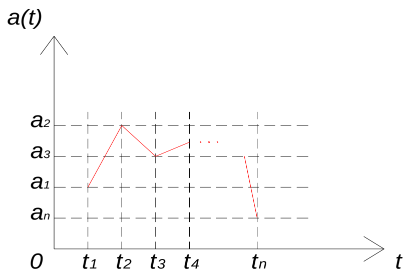


*Figure 3. Polygonaly approximated function a(t)*

The area below the polygonal line on Figure 3 is calculated by well-known trapezium formula:

$$\int_{t_1}^{t_2} a(t)dt = \frac{a_1 + a_2}{2}\cdot(t_2 - t_1) + \frac{a_2 + a_3}{2}\cdot(t_3 - t_2) + ...... + \frac{a_{n-1} + a_n}{2}\cdot(t_n - t_{n-1}) \qquad (3)$$

Formula (3) can be simplified if the counting moments are in advance periodically determined. Then $t_2 - t_1 = t_3 - t_2 = ... = t_n - t_{n-1} = \Delta t$ and the final suitable formula is given:

$$c_1\int_{t_1}^{t_2} a(t)dt = c_1\left[\frac{a_1 + a_2}{2}\cdot\Delta t + \frac{a_2 + a_3}{2}\cdot\Delta t + ...... + \frac{a_{n-1} + a_n}{2}\cdot\Delta t\right] \qquad (4)$$

DAILY MEASUREMENT EXAMPLE

On the considered parking place complex, daily payement period begins at 7:00 and ends at 21:00. The cars were counted from 7:15 till 20:30 in the intervals $\Delta t = 15\,\text{min.} = 0.25\ \text{h}$. So we get:

$a_1$ = number of parked cars at 07:15
$a_2$= number of parked cars at 07:30
$a_3$= number of parked cars at 07:45
⋮
$a_{n-1}$= number of parked cars at 20:15
$a_n$= number of parked cars at 20:30

The area shown on the Figure 3 is calculated by (4) and results are displayet in the Table 1:

| No. | TIME | CARS | ARTICLES | |
|---|---|---|---|---|
| | | $a_i$ | $\frac{a_i + a_{i+1}}{2}$ | $\frac{a_i + a_{i+1}}{2}\cdot\Delta t$ |
| 1 | 7:15 | 162 | 163 | 40.75 |
| 2 | 7:30 | 164 | 152.5 | 38.13 |
| 3 | 7:45 | 141 | 137 | 34.25 |
| 4 | 8:00 | 133 | 126 | 31.50 |

| | | | | |
|---|---|---|---|---|
| 5 | 8:15 | 119 | 117 | 29.25 |
| 6 | 8:30 | 115 | 114 | 28.50 |
| 7 | 8:45 | 113 | 109 | 21.80 |
| 8 | 9:00 | 105 | 111 | 27.75 |
| 9 | 9:15 | 117 | 118 | 29.50 |
| 10 | 9:30 | 119 | 118.5 | 29.63 |
| 11 | 9:45 | 118 | 118.5 | 29.63 |
| 12 | 10:00 | 116 | 116.5 | 29.13 |
| 13 | 10:15 | 117 | 119.5 | 29.88 |
| 14 | 10:30 | 122 | 119.5 | 29.88 |
| 15 | 10:45 | 117 | 118.5 | 29.63 |
| 16 | 11:00 | 120 | 123.5 | 30.88 |
| 17 | 11:15 | 127 | 130.5 | 32.63 |
| 18 | 11:30 | 134 | 129 | 32.25 |
| 19 | 11:45 | 124 | 128.5 | 32.13 |
| 20 | 12:00 | 133 | 132.5 | 33.13 |
| 21 | 12:15 | 132 | 133 | 33.25 |
| 22 | 12:30 | 134 | 131 | 32.75 |
| 23 | 12:45 | 128 | 131 | 32.75 |
| 24 | 13:00 | 128 | 128 | 32.00 |
| 25 | 13:15 | 131 | 130 | 32.50 |
| 26 | 13:30 | 129 | 130 | 32.50 |
| 27 | 13:45 | 131 | 132.5 | 33.13 |
| 28 | 14:00 | 134 | 135 | 33.75 |
| 29 | 14:15 | 136 | 132.5 | 33.13 |
| 30 | 14:30 | 129 | 127 | 31.75 |
| 31 | 14:45 | 125 | 125 | 31.25 |

| | | | | |
|---|---|---|---|---|
| 32 | 15:00 | 125 | 125.5 | 31.38 |
| 33 | 15:15 | 126 | 125 | 31.25 |
| 34 | 15:30 | 124 | 129 | 32.25 |
| 35 | 15:45 | 134 | 132.5 | 33.13 |
| 36 | 16:00 | 131 | 131.5 | 32.88 |
| 37 | 16:15 | 132 | 134 | 33.50 |
| 38 | 16:30 | 136 | 136.5 | 34.13 |
| 39 | 16:45 | 137 | 138.5 | 34.63 |
| 40 | 17:00 | 140 | 144.5 | 36.13 |
| 41 | 17:15 | 149 | 148 | 37.00 |
| 42 | 17:30 | 147 | 150.5 | 37.63 |
| 43 | 17:45 | 154 | 151 | 37.75 |
| 44 | 18:00 | 148 | 149 | 37.25 |
| 45 | 18:15 | 150 | 147.5 | 36.88 |
| 46 | 18:30 | 145 | 144.5 | 36.13 |
| 47 | 18:45 | 144 | 147.5 | 36.88 |
| 48 | 19:00 | 151 | 153 | 38.25 |
| 49 | 19:15 | 155 | 157.5 | 39.38 |
| 50 | 19:30 | 160 | 163 | 40.75 |
| 51 | 19:45 | 166 | 172 | 43.00 |
| 52 | 20:00 | 178 | 179 | 44.75 |
| 53 | 20:15 | 180 | 180 | 45.00 |
| 54 | 20:30 | 180 | | |
| SUMMARY | | | | **1 788.84** |

*Table 1. Summary result of daily measurement in car-hours*

The one hour parking price is 4kn≈0.53€. The total daily income is 7,155.36kn≈954.05€.

LIMITED OBSERVATION PROBLEM

Problem appears when there is impossible to overtake the whole parking complex by cameras. Then we have to estimate the number of parking vehicles in whole complex by the number of parked vehicles which are overtaken by camera. In the purpose of solving the problem, effective number of vehicles that are parked in the whole complex need to be correlated with those which are recorded by camera.

Firstly we calculate the coefficient of correlation and then we construct the linear regression formula for calculating dependence of whole number of parked cars by the number of parked cars recorded with cameras. The original data are presented in the Table 2. Considered parking complex is consisted of 193 parking places. There are 110 places overtaken by camera.

| **No.** | **Time** | **Counted number** | **Recorded number** |
|---|---|---|---|
| **n** | **t** | **Y** | **X** |
| 1 | 7:15 | 162 | 88 |
| 2 | 7:30 | 164 | 95 |
| 3 | 7:45 | 141 | 88 |
| 4 | 8:00 | 133 | 84 |
| 5 | 8:15 | 119 | 77 |
| 6 | 8:30 | 115 | 74 |
| 7 | 8:45 | 113 | 73 |
| 8 | 9:00 | 105 | 68 |
| 9 | 9:15 | 117 | 75 |
| 10 | 9:30 | 119 | 76 |
| 11 | 9:45 | 118 | 76 |
| 12 | 10:00 | 116 | 75 |
| 13 | 10:15 | 117 | 75 |
| 14 | 10:30 | 122 | 78 |
| 15 | 10:45 | 117 | 75 |
| 16 | 11:00 | 120 | 77 |
| 17 | 11:15 | 127 | 80 |

| | | | |
|---|---|---|---|
| 18 | 11:30 | 134 | 85 |
| 19 | 11:45 | 124 | 80 |
| 20 | 12:00 | 133 | 84 |
| 21 | 12:15 | 132 | 84 |
| 22 | 12:30 | 134 | 85 |
| 23 | 12:45 | 128 | 82 |
| 24 | 13:00 | 128 | 82 |
| 25 | 13:15 | 131 | 83 |
| 26 | 13:30 | 129 | 82 |
| 27 | 13:45 | 131 | 83 |
| 28 | 14:00 | 134 | 85 |
| 29 | 14:15 | 136 | 86 |
| 30 | 14:30 | 129 | 82 |
| 31 | 14:45 | 125 | 80 |
| 32 | 15:00 | 125 | 80 |
| 33 | 15:15 | 126 | 80 |
| 34 | 15:30 | 124 | 79 |
| 35 | 15:45 | 134 | 85 |
| 36 | 16:00 | 131 | 80 |
| 37 | 16:15 | 132 | 84 |
| 38 | 16:30 | 136 | 86 |
| 39 | 16:45 | 137 | 82 |
| 40 | 17:00 | 140 | 79 |
| 41 | 17:15 | 149 | 93 |
| 42 | 17:30 | 147 | 92 |
| 43 | 17:45 | 154 | 96 |
| 44 | 18:00 | 148 | 90 |

| | | | |
|---|---|---|---|
| 45 | 18:15 | 150 | 99 |
| 46 | 18:30 | 145 | 84 |
| 47 | 18:45 | 144 | 91 |
| 48 | 19:00 | 151 | 90 |
| 49 | 19:15 | 155 | 97 |
| 50 | 19:30 | 160 | 100 |
| 51 | 19:45 | 166 | 103 |
| 52 | 20:00 | 178 | 97 |
| 53 | 20:15 | 180 | 98 |
| 54 | 20:30 | 180 | 98 |
| | **Summary:** | **7345** | **4560** |

*Table 2. Comparison of parked vehicles by the way of counting*

Correlation coefficient is calculated by formula

$$r = \frac{\sum_{i=1}^{n} x_i y_i - n \cdot \bar{x} \cdot \bar{y}}{\sqrt{\left(\sum_{i=1}^{n} x_i^2 - n \cdot \bar{x}^2\right) \cdot \left(\sum_{i=1}^{n} y_i^2 - n \cdot \bar{y}^2\right)}} \quad , \tag{5}$$

Where:

$\bar{x} = 84.4$ - recorded number arithmetic mean
$\bar{y} = 136.0$ - counted number arithmetic mean
$n = 54$ - correlated pairs number.

Other values required for (5) are shown in Table 3

| No. | time | Y | X | XY | $X^2$ | $Y^2$ |
|---|---|---|---|---|---|---|
| 1 | 7:15 | 162 | 88 | 14256 | 7744 | 26244 |
| 2 | 7:30 | 164 | 95 | 15580 | 9025 | 26896 |
| 3 | 7:45 | 141 | 88 | 12408 | 7744 | 19881 |
| 4 | 8:00 | 133 | 84 | 11172 | 7056 | 17689 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 8:15 | 119 | 77 | 9163 | 5929 | 14161 |
| 6 | 8:30 | 115 | 74 | 8510 | 5476 | 13225 |
| 7 | 8:45 | 113 | 73 | 8249 | 5329 | 12769 |
| 8 | 9:00 | 105 | 68 | 7140 | 4624 | 11025 |
| 9 | 9:15 | 117 | 75 | 8775 | 5625 | 13689 |
| 10 | 9:30 | 119 | 76 | 9044 | 5776 | 14161 |
| 11 | 9:45 | 118 | 76 | 8968 | 5776 | 13924 |
| 12 | 10:00 | 116 | 75 | 8700 | 5625 | 13456 |
| 13 | 10:15 | 117 | 75 | 8775 | 5625 | 13689 |
| 14 | 10:3 | 122 | 78 | 9516 | 6084 | 14884 |
| 15 | 10:45 | 117 | 75 | 8775 | 5625 | 13689 |
| 16 | 11:00 | 120 | 77 | 9240 | 5929 | 14400 |
| 17 | 11:15 | 127 | 80 | 10160 | 6400 | 16129 |
| 18 | 11:30 | 134 | 85 | 11390 | 7225 | 17956 |
| 19 | 11:45 | 124 | 80 | 9920 | 6400 | 15376 |
| 20 | 12:00 | 133 | 84 | 11172 | 7056 | 17689 |
| 21 | 12:15 | 132 | 84 | 11088 | 7056 | 17424 |
| 22 | 12:30 | 134 | 85 | 11390 | 7225 | 17956 |
| 23 | 12:45 | 128 | 82 | 10496 | 6724 | 16384 |
| 24 | 13:00 | 128 | 82 | 10496 | 6724 | 16384 |
| 25 | 13:15 | 131 | 83 | 10873 | 6889 | 17161 |
| 26 | 13:30 | 129 | 82 | 10578 | 6724 | 16641 |
| 27 | 13:45 | 131 | 83 | 10873 | 6889 | 17161 |
| 28 | 14:00 | 134 | 85 | 11390 | 7225 | 17956 |
| 29 | 14:15 | 136 | 86 | 11696 | 7396 | 18496 |
| 30 | 14:30 | 129 | 82 | 10578 | 6724 | 16641 |
| 31 | 14:45 | 125 | 80 | 10000 | 6400 | 15625 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 32 | 15:00 | 125 | 80 | 10000 | 6400 | 15625 |
| 33 | 15:15 | 126 | 80 | 10080 | 6400 | 15876 |
| 34 | 15:30 | 124 | 79 | 9796 | 6241 | 15376 |
| 35 | 15:45 | 134 | 85 | 11390 | 7225 | 17956 |
| 36 | 16:00 | 131 | 80 | 10480 | 6400 | 17161 |
| 37 | 16:15 | 132 | 84 | 11088 | 7056 | 17424 |
| 38 | 16:30 | 136 | 86 | 11696 | 7396 | 18496 |
| 39 | 16:45 | 137 | 82 | 11234 | 6724 | 18769 |
| 40 | 17:00 | 140 | 79 | 11060 | 6241 | 19600 |
| 41 | 17:15 | 149 | 93 | 13857 | 8649 | 22201 |
| 42 | 17:30 | 147 | 92 | 13524 | 8464 | 21609 |
| 43 | 17:45 | 154 | 96 | 14784 | 9216 | 23716 |
| 44 | 18:00 | 148 | 90 | 13320 | 8100 | 21904 |
| 45 | 18:15 | 150 | 99 | 14850 | 9801 | 22500 |
| 46 | 18:30 | 145 | 84 | 12180 | 7056 | 21025 |
| 47 | 18:45 | 144 | 91 | 13104 | 8281 | 20736 |
| 48 | 19:00 | 151 | 90 | 13590 | 8100 | 22801 |
| 49 | 19:15 | 155 | 97 | 15035 | 9409 | 24025 |
| 50 | 19:30 | 160 | 100 | 16000 | 10000 | 25600 |
| 51 | 19:45 | 166 | 103 | 17098 | 10609 | 27556 |
| 52 | 20:00 | 178 | 97 | 17266 | 9409 | 31684 |
| 53 | 20:15 | 180 | 98 | 17640 | 9604 | 32400 |
| 54 | 20:30 | 180 | 98 | 17640 | 9604 | 32400 |
| **Summary:** | **Σ:** | **7345** | **4560** | **627083** | **388434** | **1015201** |

*Table 3. Values required for (5)*

Vehicles that are counted are also recorded by camera, so the recorded number of vehicles is the part of counted number of vehicles. Correlation coefficient is high: *R=0.027.*

The number of all cars parked at the complex (*Y*) is linearly dependending on the number of parked cars overtaken by camera (*X*):

$$Y = a + bX \ , \tag{6}$$

where coefficients are calculated by formulas

$$a = \frac{\sum_i y_i \cdot \sum_i x_i^2 - \sum_i x_i \cdot \sum_i x_i y_i}{n \cdot \sum_i x_i^2 - \left(\sum_i x_i\right)^2} \tag{7}$$

$$b = \frac{n \cdot \sum_i x_i y_i - \sum_i x_i \cdot \sum_i y_i}{n \cdot \sum_i x_i^2 - \left(\sum_i x_i\right)^2} \ . \tag{8}$$

Applying the values from Table 3 in (7) and (8) we get (6):

$$Y = 2.03X - 35.48 \ . \tag{9}$$

Formula (6) estimates the number of vehicles parked in whole complex by the number of vehicles (*X*) recorded by cameras. Notice that coefficient 2.03 is not equal with rate 193/110, which means that cameras overtakes those parking places that are low desired by drivers who are parking their cars.

CONCLUSION

In the first part of article we show the method for independent estimating the parking salary and it could be applied for over-sighting the companies which take money from parking places. In the second part we demonstrate the method for collecting data of parked vehicles number when oversight couldn't be completely conducted. This method is suitable for managers to control its incomes easily. The main reason not to take the rate of whole number of places and overtaken number of places is that all parking place are not equally eligible.

LITERATURE:


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