

RAF Theory Extends the Applications of GRT to the Extremely Strong Gravitational Fields

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Abstract- Modern physics is trying to solve some problems in the extremely strong gravitational field by using sophisticated methods in particle and quantum physics. But, we also should solve the mentioned problems in the classical General Relativity Theory (GRT). As it is the well-known, GRT cannot be applied to the extremely strong gravitational field. The main reason for it is an appearance of the related singularity in that field. Here we show that Relativistic Alpha Field Theory (RAFT) extends the application of GRT to the extremely strong fields including of the Planck's scale. This is the consequence of the following predictions of RAF theory: a) no a singularity at the Schwarzschild radius and b) there exist a minimal radius at $r = (GM/2c^2)$ that prevents singularity at $r = 0$, i.e. the nature protects itself. It has been theoretically proved that the metrics of RAF theory at the Schwarzschild radius, as well as at the minimal radius and at the Planck's scale are regular.

Index Terms : Relativistic alpha field theory (RAFT), No singularities in gravitational field, Extremely strong gravitational fields, Planck scale.

I. INTRODUCTION

As it is well known, General Relativity Theory (GRT) [1-6] cannot be applied to the extremely strong gravitational fields including of the Planck's scale. The main reason for this is the appearance of the related singularity in a gravitational field. Here we have used a new theory that is called Relativistic Alpha Field (RAF) theory [7,8,9]. It has been showed that RAF theory extends the capability of the GRT to the application to the extremely strong fields, including of the Planck's scale. Namely, this is the consequence of the following predictions of RAF theory: a) no a singularity at the Schwarzschild radius and b) there exist a minimal radius at $r = (GM/2c^2)$ that prevents singularity at $r = 0$, i.e. the nature protects itself.

In this paper, we started with the presentation of the solution of the field parameters in RAF theory. This solution is based on the assumption that the field parameters should connect geometry of the line element with potential energy of a particle in an alpha field. In that sense, the concept of the two dimensionless (unit less) field parameters α and α' has been introduced. These parameters are scalar functions of the potential energy of a particle in an alpha field. This new solution of the field parameters is the key point in this theory.

In order to solve the field parameters α and α' , we started with the derivation of the relative velocity of a particle in an alpha field, v_α . This relative velocity is derived from the line element in an alpha field that is given by the nondiagonal form with the Riemannian metrics. Thus, the relative velocity

of a particle in an alpha field, v_α , is described as the function of the field parameters α and α' and a particle velocity v in the total vacuum (without any potential field). This structure of the relative velocity v_α directly connects the line elements of the Special and General Relativity. Namely, in the case of the total vacuum (without any potential field), field parameters α and α' become equal to unity and, consequently, the relative velocity v_α becomes equal to the particle velocity v in the total vacuum. This is a direct transition of the line element from the General to the Special Theory of Relativity.

The main point in this paper is the theoretical confirmation that Relativistic Alpha Field Theory (RAFT) really can extend the application of GRT to the extremely strong gravitational fields including of the Planck's scale. In that sense, it has been presented that the metrics at the Schwarzschild radius as well as at the minimal radius are regular. Further, the minimal radius of the Planck's mass is derived, which is equal to the half of the Planck's length. Thus, the Planck's length is a diameter of the Planck's mass. The metric at the Planck's scale is also regular.

This paper is organized as follows. In Sec. II, we show derivation of the relative velocity of a particle in an alpha field v_α as the function of the field parameters α and α' . Solution of the field parameters α and α' in a general form, as the function of the potential energy U is presented in Sec. III. Solution of the field parameters α and α' in gravitational field is considered in Sec. IV. Derivation of energy-momentum tensor for gravitational field is pointed out in Sec. V. The theoretical proofs that RAF theory extends applications of GRT to the extremely strong gravitational field are presented in Sec. VI. Finally, the related conclusion and the reference list are presented in Sec. VII and Sec. VIII, respectively.

II. DERIVATION OF RELATIVE VELOCITY v_α

RAF theory is based on the following two definitions [7]:

Definition 1. An alpha field is a potential field that can be described by two dimensionless (unit less) scalar parameters α and α' . To this category belong, among the others, electrical and gravitational fields.

Definition 2. Field parameters α and α' are described as the scalar dimensionless (unit less) functions of the potential energy U of a particle in an alpha field.

In order to solve the field parameters α and α' , we started with the derivation of the relative velocity of a particle in an alpha field, v_α .

Proposition 1. If the line element in an alpha field is defined by the nondiagonal form with the Riemannian metrics

[10-13]:

$$ds^2 = -\alpha\alpha'c^2dt^2 - \kappa(\alpha - \alpha')_x cdt dt - \kappa(\alpha - \alpha')_y cdt dy - \kappa(\alpha - \alpha')_z cdt dz + dx^2 + dy^2 + dz^2, \quad (1)$$

then the relative velocity of a particle in an alpha field, v_α , can be described as the function of the field parameters α and α'

$$v_\alpha = v - \frac{\kappa(\alpha - \alpha')c}{2}. \quad (2)$$

In the previous equation v is a particle velocity in the total vacuum (without any potential field), c is the speed of the light in a vacuum and κ is a constant.

Proof if the Proposition 1. This proof has been presented in the reference [7].

Proposition 2. The relation in (2) satisfies the well-known condition [1, 2, 11-13] for the metric tensor of the line element (1)

$$\sqrt{-det(g_{\mu\nu})} = 1, \rightarrow \kappa = \pm 1. \quad (3)$$

Proof if the Proposition 2. This proof has been presented in the reference [7].

Proposition 3. Let $d\tau$ and dt are differentials of the proper time and coordinate time (respectively) of the moving particle in an alpha field. Further, let H is a transformation factor, as an invariant of an alpha field, and v_α is a particle velocity in that field given by (2). For that case, the transformation factor H has the following form

$$H = \frac{dt}{d\tau} = \left(1 - \frac{v_\alpha^2}{c^2}\right)^{-1/2} = \left(\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa(\alpha - \alpha')cv}{c^2}\right)^{-1/2}. \quad (4)$$

Proof of the Proposition 3. This proof has been presented in the reference [7].

Furthermore, if a particle is moving in a total vacuum (without any potential field), then we have $\alpha = \alpha' = 1$, and the relation (4) is transformed into the transformation factor γ valid in the Special Relativity

$$\alpha = \alpha' = 1, \rightarrow H = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \gamma = \frac{dt}{dt'}, \quad (5)$$

$$\rightarrow d\tau = dt'.$$

III. SOLUTION OF THE FIELD PARAMETERS IN AN ALPHA FIELD

Proposition 4. Let m_0 is a rest mass of a particle, U is a potential energy of a particle in an alpha field, c is the speed of the light in a vacuum and (i) is an imaginary unit. In that case the field parameters α and α' can be described as dimensionless (unit less) functions of the potential energy U of a particle in an alpha field. There are four solutions for both parameters α and α' in an alpha field that can be presented by the following relations:

$$f(U) = 2U / m_0c^2 + \left(U / m_0c^2\right)^2, \rightarrow \alpha_1 = 1 + i\sqrt{f(U)},$$

$$\alpha'_1 = 1 - i\sqrt{f(U)}, \alpha_2 = 1 - i\sqrt{f(U)}, \alpha'_2 = 1 + i\sqrt{f(U)},$$

$$\alpha_3 = -1 + i\sqrt{f(U)}, \alpha'_3 = -1 - i\sqrt{f(U)},$$

$$\alpha_4 = -1 - i\sqrt{f(U)}, \alpha'_4 = -1 + i\sqrt{f(U)}. \quad (6)$$

Proof of the Proposition 4. This proof has been presented in the reference [7].

The four solutions of the field parameter α in (6) can be presented in the form

$$\alpha_{1,2} = 1 \pm i \sqrt{\frac{2U}{m_0c^2} + \left(\frac{U}{m_0c^2}\right)^2}, \quad (7)$$

$$\alpha_{3,4} = -1 \pm i \sqrt{\frac{2U}{m_0c^2} + \left(\frac{U}{m_0c^2}\right)^2}.$$

The related four solutions of the field parameter α' in (6) can be presented with the following relations

$$\alpha'_{1,2} = 1 \mp i \sqrt{\frac{2U}{m_0c^2} + \left(\frac{U}{m_0c^2}\right)^2}, \quad (8)$$

$$\alpha'_{3,4} = -1 \mp i \sqrt{\frac{2U}{m_0c^2} + \left(\frac{U}{m_0c^2}\right)^2}.$$

Thus, the four solutions of the field parameters α and α' can be obtained by the unification of the two parameter structures given by (7) and (8):

$$f(U) = 2U / m_0c^2 + \left(U / m_0c^2\right)^2, \rightarrow$$

$$\alpha_{1,2} = 1 \pm i\sqrt{f(U)}, \alpha'_{1,2} = 1 \mp i\sqrt{f(U)}, \quad (9)$$

$$\alpha_{3,4} = -1 \pm i\sqrt{f(U)}, \alpha'_{3,4} = -1 \mp i\sqrt{f(U)}.$$

Further, it is easy to prove that all $\alpha_i\alpha'_i$ pairs from (9) are creating an invariant $\alpha\alpha'$

$$\alpha_i\alpha'_i = \left(1 + \frac{U}{m_0c^2}\right)^2 = \alpha\alpha', \quad i = 1, 2, 3, 4. \quad (10)$$

For calculation some of the quantities in an alpha field we often need to know the difference of the field parameters ($\alpha - \alpha'$):

$$\alpha_1 - \alpha'_1 = 2i\sqrt{f(U)}, \quad \alpha_2 - \alpha'_2 = -2i\sqrt{f(U)},$$

$$\alpha_3 - \alpha'_3 = 2i\sqrt{f(U)}, \quad \alpha_4 - \alpha'_4 = -2i\sqrt{f(U)}, \quad (11)$$

$$(\alpha_1 - \alpha'_1) = (\alpha_3 - \alpha'_3), \quad (\alpha_2 - \alpha'_2) = (\alpha_4 - \alpha'_4).$$

The obtained relations in (9), (10) and (11) are valid generally and for their calculation we only need to know potential energy U of the particle in the related potential field.

Remarks 1. From the equations (9), (10) and (11) we can see that there are three very important properties of the

solutions of the field parameters α and α' : a) parameters α and α' are dimensionless (unitless) field parameters, b) there are four solutions of the field parameters α and α' that reminds us to the Dirac's theory [14] and c) the quantity $\alpha\alpha'$ is an invariant related to the four solutions of the field parameters α and α' .

IV. SOLUTION OF THE FIELD PARAMETERS IN GRAVITATIONAL FIELD

If a particle with the rest mass m_0 is in a gravitational field, then the potential energy of the particle in that field U_g is described by the well-known relation [1-6]

$$U_g = m_0 V_g = m_0 A_{g0} = -\frac{m_0 GM}{r}. \quad (12)$$

Here $V_g = A_{g0}$ is a scalar potential of the gravitational field, G is the gravitational constant, M is a gravitational mass and r is a gravitational radius. The four solutions of the field parameters α and α' for the particle in a gravitational field can be obtained by the substitution of the potential energy U_g from (12) into the general relations in (9):

$$i\sqrt{f(U_g)} = -\sqrt{2GM/rc^2 - (GM/rc^2)^2} = -\sqrt{\cdot}, \rightarrow$$

$$\alpha_1 = 1 - \sqrt{\cdot}, \quad \alpha'_1 = 1 + \sqrt{\cdot}, \quad \alpha_2 = \alpha'_1, \quad \alpha'_2 = \alpha_1,$$

$$\alpha_3 = -1 - \sqrt{\cdot}, \quad \alpha'_3 = -1 + \sqrt{\cdot}, \quad \alpha_4 = \alpha'_3, \quad \alpha'_4 = \alpha_3,$$

$$GM \ll rc^2, \rightarrow (GM/rc^2)^2 \cong 0, \rightarrow i\sqrt{f(U_g)} = -\sqrt{2GM/rc^2}. \quad (13)$$

The first three lines in equations (13) describe a strong gravitational field. If the quadratic term $(GM/rc^2)^2 \approx 0$ then the field parameters (13) describe a weak gravitational field as we have in our solar system. The differences of the field parameters ($\alpha - \alpha'$) for a particle in a gravitational field have the forms:

$$\alpha_1 - \alpha'_1 = -2\sqrt{\frac{2GM}{rc^2} - \left(\frac{GM}{rc^2}\right)^2}, \quad \alpha_3 - \alpha'_3 = (\alpha_1 - \alpha'_1),$$

$$\alpha_2 - \alpha'_2 = 2\sqrt{\frac{2GM}{rc^2} - \left(\frac{GM}{rc^2}\right)^2}, \quad \alpha_4 - \alpha'_4 = (\alpha_2 - \alpha'_2). \quad (14)$$

Remarks 2. In the references [8,9,15] it has been shown that field parameters (13) and (14) satisfy the Einstein's field equations with a cosmological constant $\Lambda = 0$. In the case of a strong static gravitational field [16-20], the quadratic term $(GM/rc^2)^2$ in (13) and (14) generates the related energy-momentum tensor $T_{\mu\eta}$ for the static field. For that case, we do not need to add by hand the related energy-momentum tensor $T_{\mu\eta}$ on the right side of the Einstein's field equations.

The second interpretation could be that the quadratic term $(GM/rc^2)^2$ generates the cosmological parameter Λ as a function of a gravitational radius [21] for $T_{\mu\eta} = 0$. It has been shown [22] that this solution of Λ is valid for both Planck's and cosmological scales. In the case of a weak static

gravitational field, like in our solar system, the field parameters (13) satisfy the Einstein's field equations in a vacuum ($T_{\mu\eta} = 0, \Lambda = 0$). The general metrics of the relativistic alpha field theory [10] has been applied to the derivation of dynamic model of nanorobot motion in multipotential field [23].

V. ENERGY-MOMENTUM TENSOR FOR GRAVITATIONAL FIELD

The basic problem of this section is to determine the energy-momentum tensors for gravitational field in the Einstein's four-dimensional space-time (4D). In that sense, we started with the general line element ds^2 given by the relation (1). Following the well-known procedure [1-6], this line element can be transformed into the spherical polar coordinates in the nondiagonal form

$$ds^2 = -\alpha\alpha' c^2 dt^2 - \kappa(\alpha - \alpha') c dt dr + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (15)$$

The line element (15) belongs to the well-known form of the Riemann's type line element [10-13]

$$ds^2 = g_{00} (dx^0)^2 + 2g_{01} dx^0 dx^1 + g_{11} (dx^1)^2 + g_{22} (dx^2)^2 + g_{33} (dx^3)^2. \quad (16)$$

Comparing the equations (15) and (16) we obtain the coordinates and components of the covariant metric tensor, valid for the line element (15):

$$dx^0 = c dt, \quad dx^1 = dr, \quad dx^2 = d\theta, \quad dx^3 = d\phi,$$

$$g_{00} = -\alpha\alpha', \quad g_{01} = g_{10} = \frac{-\kappa(\alpha - \alpha')}{2}, \quad (17)$$

$$g_{11} = 1, \quad g_{22} = r^2, \quad g_{33} = r^2 \sin^2 \theta.$$

Starting with the line element (15) we employ, for the convenient, the following substitutions:

$$v = \alpha\alpha', \quad \lambda = \kappa(\alpha' - \alpha) / 2. \quad (18)$$

In that case the nondiagonal line element (15) is transformed into the new relation

$$ds^2 = -v c^2 dt^2 + 2\lambda c dt dr + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (19)$$

Using the coordinate system (17), the related covariant metric tensor $g_{\mu\eta}$ of the line element (19) is presented by the matrix form

$$[g_{\mu\eta}] = \begin{bmatrix} -v & \lambda & 0 & 0 \\ \lambda & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{bmatrix}. \quad (20)$$

This tensor is symmetric and has six non-zero elements as we expected that should be. The contravariant metric tensor $g^{\mu\eta}$

of the nondiagonal line element (19), is derived by inversion of the covariant one (20)

$$[g^{\mu\eta}] = \begin{bmatrix} -1/(v+\lambda^2) & \lambda/(v+\lambda^2) & 0 & 0 \\ \lambda/(v+\lambda^2) & v/(v+\lambda^2) & 0 & 0 \\ 0 & 0 & 1/r^2 & 0 \\ 0 & 0 & 0 & 1/r^2 \sin^2 \theta \end{bmatrix}. \quad (21)$$

The determinants of the tensors (20) and (21) are given by the relations:

$$\det[g_{\mu\eta}] = -r^4 (v + \lambda^2) \sin^2 \theta, \quad (22)$$

$$\det[g^{\mu\eta}] = - \left(\frac{1}{r^4 (v + \lambda^2) \sin^2 \theta} \right).$$

Proposition 5. If the gravitational static field is described by the line element (19), then the solution of the Einstein field equations gives the energy momentum tensor $T_{\mu\eta}$ of that field in the following form

$$T_{\mu\eta} = (T_{00}, T_{01}, T_{10}, T_{11}, T_{22}, T_{33}) \quad (23)$$

$$= (v, -\lambda, -\lambda, -1, r^2, r^2 \sin^2 \theta) \frac{(GM)^2}{8\pi G r^4}.$$

Here G and M are the gravitational constant and the gravitational mass, respectively.

Proof of the proposition 5. In order to prove of the proposition 5, we can start with the second type of the Christoffel symbols of the metric tensors (20) and (21). These symbols can be calculated by employing the well-known relation [1-6]

$$\Gamma_{\eta\mu}^\gamma = \frac{g^{\kappa\gamma}}{2} [g_{\kappa\eta,\mu} + g_{\kappa\mu,\eta} - g_{\eta\mu,\kappa}], \quad \kappa, \gamma, \eta, \mu = 0, 1, 2, 3. \quad (24)$$

Thus, employing (19), (20), (21) and (24), we obtain the second type Christoffel symbols of the spherically symmetric non-rotating body:

$$\Gamma_{00}^0 = (\dot{v} + 2\lambda\dot{\lambda} + v'\lambda) / D, \quad \Gamma_{01}^0 = \Gamma_{10}^0 = v' / D, \quad \Gamma_{11}^0 = -2\lambda' / D,$$

$$\Gamma_{22}^0 = -2\lambda r / D, \quad \Gamma_{33}^0 = (-2\lambda r \sin^2 \theta) / D, \quad \Gamma_{00}^1 = (-\dot{v}\lambda + 2v\dot{\lambda} + vv') / D,$$

$$\Gamma_{01}^1 = \Gamma_{10}^1 = -v'\lambda / D, \quad \Gamma_{11}^1 = 2\lambda'\lambda / D, \quad \Gamma_{22}^1 = -2vr / D,$$

$$\Gamma_{33}^1 = (-2vr \sin^2 \theta) / D, \quad \Gamma_{12}^2 = \Gamma_{21}^2 = \frac{1}{r}, \quad \Gamma_{33}^2 = -\sin\theta \cos\theta, \quad \Gamma_{13}^3 = \Gamma_{31}^3 = \frac{1}{r},$$

$$\Gamma_{23}^3 = \Gamma_{32}^3 = ctg\theta, \quad D = 2(v + \lambda^2), \quad \frac{\partial v}{\partial t} = \dot{v}, \quad \frac{\partial v}{\partial r} = v', \quad \frac{\partial \lambda}{\partial t} = \dot{\lambda}, \quad \frac{\partial \lambda}{\partial r} = \lambda'. \quad (25)$$

For a static field, the Christoffel symbols Γ_{00}^0 and Γ_{00}^1 are reduced to the simplest form:

$$\Gamma_{00}^0 = \frac{v'\lambda}{2(v + \lambda^2)}, \quad \Gamma_{00}^1 = \frac{vv'}{2(v + \lambda^2)}, \quad \frac{\partial v}{\partial r} = v', \quad \frac{\partial \lambda}{\partial r} = \lambda'. \quad (26)$$

In a static field, the other Christoffel symbols in (25) are remaining unchanged.

As it is well known, the determinant of the metric tensor of the line element (19) should satisfy the following condition [1-6, 11-13]

$$\sqrt{-\det[g_{\mu\eta}]} = \sqrt{r^4 (v + \lambda^2) \sin^2 \theta} = 1. \quad (27)$$

Including the normalization of the radius, $r = 1$, and the angle $\theta = 90^\circ$ in (27) we obtain the important relations between the parameters v and λ :

$$v + \lambda^2 = 1, \quad v = 1 - \lambda^2, \quad (28)$$

$$v' = -2\lambda\lambda', \quad v'' = -2(\lambda'^2 + \lambda\lambda'').$$

If we take into account the relations (28), then the Christoffel symbols in (25) and (26) become the only functions of the parameter λ .

For calculation of the related components of the Riemannian tensor $R_{\beta\eta\mu}^\kappa$ and Ricci tensor $R_{\eta\mu}$ of the line element (19) we can employ the following relations [1-6]:

$$R_{\beta\eta\mu}^\kappa = \Gamma_{\beta\mu,\eta}^\kappa - \Gamma_{\beta\eta,\mu}^\kappa + \Gamma_{\sigma\eta}^\kappa \Gamma_{\beta\mu}^\sigma - \Gamma_{\sigma\mu}^\kappa \Gamma_{\beta\eta}^\sigma, \quad (29)$$

$$R_{\eta\mu} = R_{\eta\kappa\mu}^\kappa = R_{\mu\eta}, \quad \kappa, \beta, \eta, \mu, \sigma = 0, 1, 2, 3.$$

Applying the Christoffel symbols (25) to the relations (29) we obtain the related Ricci tensor for the static field of the line element (19), with the following components:

$$R_{00} = -(1 - \lambda^2) \left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r} \right),$$

$$R_{01} = R_{10} = -\lambda \left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r} \right), \quad (30)$$

$$R_{11} = \left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r} \right), \quad R_{22} = 2\lambda\lambda' r + \lambda^2,$$

$$R_{33} = \left(2\lambda\lambda' r + \lambda^2 \right) \sin^2 \theta.$$

The other components of the Ricci tensor are equal to zero. The related Ricci scalar for the static field is determined by the equation

$$R = g^{\mu\eta} R_{\mu\eta}, \quad \mu, \eta = 0, 1, 2, 3, \rightarrow$$

$$R = 2 \left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r} \right) + 2 \left(\frac{2\lambda\lambda'}{r} + \frac{\lambda^2}{r^2} \right). \quad (31)$$

In order to calculate the energy-momentum tensor $T_{\mu\eta}$ for the static field, one should employ Ricci tensor (30), Ricci scalar (31) and the Einstein's field equations [1-6] without a cosmological constant ($\Lambda = 0$)

$$R_{\mu\eta} - \frac{1}{2} g_{\mu\eta} R = kT_{\mu\eta}, \quad k = \frac{8\pi G}{c^4}, \quad \mu, \eta = 0, 1, 2, 3. \quad (32)$$

Here G is the Newton's gravitational constant, c is the speed of the light in a vacuum and $T_{\mu\eta}$ is the energy-momentum tensor. Thus, employing the Einstein's field equations (32) we obtain the following relations for calculation of the components of the energy-momentum tensor $T_{\mu\eta}$:

$$\begin{aligned} kT_{00} &= (1-\lambda^2) \left(\frac{2\lambda\lambda'}{r} + \frac{\lambda^2}{r^2} \right), \quad kT_{01} = kT_{10} = -\lambda \left(\frac{2\lambda\lambda'}{r} + \frac{\lambda^2}{r^2} \right), \\ kT_{11} &= (-1) \left(\frac{2\lambda\lambda'}{r} + \frac{\lambda^2}{r^2} \right), \quad kT_{22} = (-r^2) \left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r} \right), \\ kT_{33} &= (-r^2 \sin^2 \theta) \left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r} \right), \quad k = \frac{8\pi G}{c^4}. \end{aligned} \quad (33)$$

For calculation of the components of the energy-momentum tensor, $T_{\mu\eta}$, by the relations (33) we should know the parameter λ and its derivations λ' and λ'' for the related static field. Parameter λ is defined by (18) as the function of the field parameters α and α'

$$\lambda = \kappa(\alpha' - \alpha) / 2 = -\kappa(\alpha - \alpha') / 2, \quad \kappa = \pm 1. \quad (34)$$

Applying of the solution of the field parameters α and α' in a gravitational field (13,14) to the relation (34) we obtain the two solutions of the parameter λ in a gravitational static field

$$\lambda = \mp \kappa \sqrt{\frac{2GM}{rc^2} - \left(\frac{GM}{rc^2} \right)^2}. \quad (35)$$

Now, one can calculate of the all components needed for determination of the energy-momentum tensor $T_{\mu\eta}$ in a static gravitational field:

$$\begin{aligned} \lambda' &= \mp \kappa \left[\frac{GM}{r^2 c^2} - \left(\frac{GM}{r^2 c^2} \right) \left(\frac{GM}{rc^2} \right) \right] / \sqrt{\frac{2GM}{rc^2} - \left(\frac{GM}{rc^2} \right)^2}, \\ \lambda\lambda' &= \left[-\frac{GM}{r^2 c^2} + \left(\frac{GM}{r^2 c^2} \right) \left(\frac{GM}{rc^2} \right) \right], \\ \frac{2\lambda\lambda'}{r} &= 2 \left[-\frac{GM}{r^3 c^2} + \left(\frac{GM}{r^2 c^2} \right)^2 \right], \quad \frac{\lambda^2}{r^2} = \left[\frac{2GM}{r^3 c^2} - \left(\frac{GM}{r^2 c^2} \right)^2 \right], \\ \lambda'^2 + \lambda\lambda'' &= (\lambda\lambda')' = \frac{2GM}{r^3 c^2} - 3 \left(\frac{GM}{r^2 c^2} \right)^2. \end{aligned} \quad (36)$$

Applying the relations (36) to the equations (31) and (33) we obtain the components of the energy-momentum tensor and Ricci scalar valid for the static gravitational field:

$$\begin{aligned} kT_{00} &= (1-\lambda^2) \left(\frac{GM}{r^2 c^2} \right)^2 = v \left(\frac{GM}{r^2 c^2} \right)^2, \\ kT_{01} &= kT_{10} = -\lambda \left(\frac{GM}{r^2 c^2} \right)^2, \quad k = \frac{8\pi G}{c^4}, \\ kT_{11} &= (-1) \left(\frac{GM}{r^2 c^2} \right)^2, \quad kT_{22} = r^2 \left(\frac{GM}{r^2 c^2} \right)^2, \\ kT_{33} &= (r^2 \sin^2 \theta) \left(\frac{GM}{r^2 c^2} \right)^2, \\ R &= 2 \left(\lambda'^2 + \lambda\lambda'' + \frac{2\lambda\lambda'}{r} \right) + 2 \left(\frac{2\lambda\lambda'}{r} + \frac{\lambda^2}{r^2} \right) \\ &= -2 \left(\frac{GM}{r^2 c^2} \right)^2 + 2 \left(\frac{GM}{r^2 c^2} \right)^2 = 0. \end{aligned} \quad (37)$$

From the previous relations we can see that the Ricci scalar is equal to zero. Finally, included parameter k into the relations (37), we obtain the components of the energy-momentum tensor in the static gravitational field

$$\begin{aligned} T_{\mu\eta} &= [T_{00}, T_{01}, T_{10}, T_{11}, T_{22}, T_{33}] \\ &= \left[v, -\lambda, -\lambda, -1, r^2, r^2 \sin^2 \theta \right] \frac{(GM)^2}{8\pi G r^4}. \end{aligned} \quad (38)$$

Because the relation (38) is equal to the relation (23), the proof of the proposition 5 is finished.

VI. PROOFS THAT RAF THEORY EXTENDS APPLICATIONS OF GRT TO EXSTREMELY STRONG GRAVITATIONAL FIELD

As it is the well known gravitational fields become strong and extremely strong at the Schwarzschild and smaller radiuses. In order to extend the application of GRT to the extremely strong gravitational fields we have to have the related theory with regular line element in that region. Just RAF theory offers regular line element in that region by the following predictions: a) no a singularity at the Schwarzschild radius and b) there exists a minimal radius at the position $r = r_{min} = GM / 2c^2$ that prevents singularity at $r = 0$, i.e. the nature protects itself. Thus, RAF theory has a regular line element in the region $r_{min} \leq r \leq \infty$.

In order to prove predictions a) and b) we start with the solution of the parameters v and λ in a static gravitational field given by (28) and (35) and valid for the line element (19):

$$\lambda = \mp \kappa \sqrt{\frac{2GM}{rc^2} \left(1 - \frac{GM}{2rc^2}\right)}, \quad \kappa = \pm 1,$$

$$v = 1 - \lambda^2 = 1 - \frac{2GM}{rc^2} + \left(\frac{GM}{rc^2}\right)^2 = \left(1 - \frac{GM}{rc^2}\right)^2,$$

$$r_{sch} = \frac{2GM}{c^2}, \quad \rightarrow v_{sch} = \frac{1}{4}, \quad \lambda_{sch} = \mp \kappa \sqrt{\frac{3}{4}}, \quad (39)$$

$$r_{min} = \frac{GM}{2c^2}, \quad \rightarrow v = 1, \quad \lambda = 0, \quad r < r_{min} \rightarrow \lambda = \lambda_{im},$$

$$r = \frac{GM}{c^2}, \quad \rightarrow v = 0, \lambda = \mp \kappa, \quad r \rightarrow \infty, \quad v \rightarrow 1, \quad \lambda \rightarrow 0,$$

The related line elements are given by the relations:

$$ds_{r_{sch}}^2 = -\frac{1}{4}c^2 dt^2 \mp 2k\sqrt{\frac{3}{4}}cdt dr + dr^2 + r^2 d\Omega^2,$$

$$ds_{r_{min}}^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2,$$

$$ds_{r=GM/c^2}^2 = \mp kc dt dr + dr^2 + r^2 d\Omega^2, \quad (40)$$

$$ds_{r \rightarrow \infty}^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2,$$

$$d\Omega = r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad k = \pm 1.$$

Following the relations (39) and (40) we can see that at the Schwarzschild radius, r_{sch} , parameters v and λ are regular. This proves the prediction a) no a singularity at the Schwarzschild radius. Further, from the same relations, we also can see that at the minimal radius $r = r_{min} = GM/2c^2$ parameters v and λ are also regular and for $r < r_{min}$ parameter λ becomes imaginary number $\lambda = \lambda_{im}$. This proves the prediction b) there exists a minimal radius at $r = r_{min} = GM/2c^2$ that prevents singularity at $r=0$. It seems that the existence of the minimal radius tells us that the nature protect itself from the singularity. Thus, we can say that the metrics of the line element (19) is regular for a gravitational field in the region $r_{min} \leq r \leq \infty$. On that way, the proof of the propositions a) and b) is finished. At the same time, it has been proved that RAF theory extends the applications of GRT to the extremely strong gravitational fields.

Now we can assume that the Planck's mass M_p [26] is the spherically symmetric non-rotating body. For that case one can calculate the minimal radius of the Planck's mass, r_{pm} :

$$r_{pm} = \frac{GM_p}{2c^2} \rightarrow \frac{M_p}{2r_{pm}} = \frac{c^2}{G},$$

$$\frac{M_p}{L_p} = \frac{\sqrt{\hbar c/G}}{\sqrt{\hbar G/c^3}} = \frac{c^2}{G}, \quad \rightarrow 2r_{pm} = L_p \rightarrow r_{pm} = \frac{L_p}{2}. \quad (41)$$

By the relations in (41) we found out that the minimal radius of the Planck's mass M_p is equal to half of the Planck's length [27]. This means that Planck's length is the diameter of the Planck's mass. At the minimal radius of the Planck's

mass, $r_{pm} = L_p/2$, parameters v and λ and the related line element have the following forms:

$$v = \left(1 - \frac{GM_p}{(L_p/2)c^2}\right)^2 = \left(1 - 2\left(\frac{G}{c^2}\right)\left(\frac{c^2}{G}\right)\right)^2 = 1,$$

$$\lambda = \mp \kappa \sqrt{\frac{2GM_p}{(L_p/2)c^2} \left(1 - \frac{GM_p}{2(L_p/2)c^2}\right)}, \quad \kappa = \pm 1,$$

$$\lambda = \mp \kappa \sqrt{4\left(\frac{G}{c^2}\right)\left(\frac{c^2}{G}\right)\left(1 - \left(\frac{G}{c^2}\right)\left(\frac{c^2}{G}\right)\right)} = 0, \quad (42)$$

$$ds_{r_{pm}}^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2.$$

From the relations in (42), we can see that at the minimal radius $r_{pm} = L_p/2$ of the Planck's mass parameters v and λ , as well as the line element are also regular. This proves prediction a) no a singularity at the minimal radius of the Planck's mass. If the radius is less than the minimal radius $r_{pm} = L_p/2$, then parameter λ for the Planck's mass becomes imaginary number $\lambda = \lambda_{im}$. This proves the prediction b) there exists a minimal radius of the Planck's mass $r_{pm} = L_p/2$ that prevents singularity at $r=0$. It means that the existence of the minimal radius tells us that the nature protect itself from the singularity. Thus, we can say that the metrics of the line element in (19) is also regular for a gravitational field at the Planck's scale. On that way, the proofs of the propositions a) and b) at the Planck's scale are finished. At the same time, it has been proved that RAF theory extends the applications of GRT to the extremely strong gravitational fields at the Planck's scale.

VII. CONCLUSION

In this paper, we show that Relativistic Alpha Field Theory (RAFT) extends the application of GRT to the extremely strong gravitational fields including of the Planck's scale. It has been presented that the metrics at the Schwarzschild radius as well as at the minimal radius are regular. Further, the minimal radius of the Planck's mass is derived, which is equal to the half of the Planck's length. Thus, the Planck's length is a diameter of the Planck's mass. The metric at the Planck's scale is also regular. The presented results are the consequence of the very important predictions of RAF theory: a) no a singularity at the Schwarzschild radius and b) there exists a minimal radius at $r = r_{min} = GM/2c^2$ that prevents singularity at $r=0$, i.e. the nature protects itself. If the predictions of RAF theory are correct, then it could give the new light to the regions like black holes, quantum theory, high energy physics, Big Bang theory and cosmology.

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