A brief (historical) review on definitions and applications of the geoid

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Motivation and introduction

Bjerhammar (1969): Theory of a new geoid

"The geoid is one of the fundamental concepts."

"It is strange to find that many different kinds of geoids are used without distinguishing between the basic differences and it is also confusing to find that different geoids are superimposed or linked together without an adequate definition of the principal differences. "

Now more than ever it is important to pay special attention to the differences.

"Few" variations of the geoid found in the literature

- geoid, quasi-geoid
- gravimetric, hybrid (MSL)
- satellite-only, terrestrial only, astrogeodetic, gravimetric, GNSS/levelling, ... and all sorts of mutual combinations
- mean tide, non-tidal, zero-tide
- co-geoid, free-air, isostatic, RTM, Rudzki, Molodensky
- wavelengths
- relativistic *a*-geoid or *u* geoid

Definitions of the geoid (I)

Gauss-Listing (*classical*)

Gauss (1828). "Geometric surface of the Earth which intersects the direction of gravity at right angles and from which the surface of the world ocean is a part."

Listing (1873) geoid is... "a mathematical surface of the earth"... and ... "the best approximation of the MSL."

Nakiboglu (1979). "Non-hydrostatic geoid is the shape that the Earth should have if gravity and rotation were in equilibrium."

Torge (1993). "About 2/3 of the Earth's surface (oceans) approximately coincide with the geoid, the rest (continents) can be easily referred to it by leveling and gravity measurements. "

Sjöberg & Bagherbandi (2017) "The geoid is the equipotential surface that most closely coincides with the undisturbed mean sea level (and its continuation through the continents). Disturbances are caused by ocean tides, streams, winds, variations in salinity and temperature, etc, of the order of $\pm$ 2 m."

More variations in: Heiskanen & Moritz (1967), Hotine (1969),Vaníček & Martinec (1994), Vaníček & Krakiwsky (2015).

Definitions of the geoid (II)

Gauss-Listing (conventional)

Sánchez et al. (2016). "Since the Earth's gravity potential field contains an infinite number of equipotential surfaces, the geoid is to be defined arbitrarily by convention."(...)
Geoid is: "the potential value obtained for the epoch 2010.0 (W_0=62,636,853.4 m^2s^{-2}) recommended as the present best estimate for the W_0 value." **Also in: Petit, Gérard and Luzum, Brian (2010).**

Relativistic

Einstein (1950), Bjerhammar (1962). "In relativistic geodesy, the geoid is the surface nearest to MSL on which precise clocks run with the same speed."

Philipp et al. (2017) We present a definition of the geoid that is based on the formalism of general relativity without approximations; i.e. it allows for arbitrarily strong gravitational fields. **Also in: Jekeli & Kwon (2002)**

Geoid is (or should be) completely described by...

Necessary parameters

- constant potential W_0
- reference ellipsoid
- terrestrial reference frame
- epoch
- geocentric gravitational constant GM
- tidal system
- density of topographic masses

Desirable parameters

- error estimates for geoid heights σ_N
- geoid vertical velocity $N^{\mathbf{dot}}$
- error estimates of velocities $\sigma_{N^{\mathbf{dot}}}$

Supplementary metadata

- boundaries
- grid interval $(\Delta\varphi, \Delta\lambda)$
- node (point/mean, center/grid)

Reference: **Smith et al. (2011)**

Applications of geoid models

Geodetic

- reference surface for leveling
- vertical datum for orthometric heights
- transformation of ellipsoidal into orthometric heights
- local and regional vertical datum unification
- satellites orbits prediction

Geophysics

- studies of the Earth's structure
- vertical crustal movements
- plate tectonics and seismics
- locating deposits of ores, oil and gas

Other

- hydrographic surveying
- airplane and marine navigation
- sea level rise, floods
- subsidence

Actual geodetic application: gravimetric geoid-based HRS

Traditional HRS

- vertical datum: mean sea level at tide gauge(s)
- vertical reference frame: levelling
- preferably: gravimetry
- height system: orthometric or normal (-orthometric)
- compatibility with GNSS: fit gravimetric geoid to levelling

New possibility for HRS

- vertical datum: geoid
- vertical reference frame: geoid
- height system: orthometric
- quality control on monuments

Recent efforts

- Already adopted: Canada, New Zealand, Indonesia
- Currently working: USA, Australia, Japan
- Considering: Turkey, Sweden, Malaysia, Tanzania, India, South African Republic, Mexico, Croatia

Accuracy of regional geoid models

Civilian and scientific requirements for geoid model **accuracy**...

- purely geometric ellipsoidal (GNSS) heights in the next decade will be obtainable with *sub-cm* (even in real-time)
- geophysics, oceanography, ...

The progress in accuracy

- 70's-90's: metres
- 90's-00's: decimetres
- 00's-2020's: centimetres
- 2020- ...: *sub*-centimetres

So now...

Almost every source and type of error, which was considered small or ignorable in the past, becomes important and significant.

Some problems in reaching *sub*-cm geoid model (I)

I. Conventions and recommendations

- normal field, tidal system, W_0 and time epoch, height system, gravity reference system, time changes (static vs dynamic)

IAG 0.1.2 Strategy for the Realization of the International Height Reference System (IHRS) (Sánchez, L.)

II. Theoretical

- Earth's structure and complete density information will never be completely known.
- Integral equations are replaced by approximative equations where simplicity and efficiency is gained at the cost of accuracy.

ICCT JSG 0.15: Regional geoid/quasi- geoid modelling – Theoretical framework for the sub-centimetre accuracy (Huang, J.)

Some problems in reaching *sub*-cm geoid model (II)

III. Data

- requirements on the quantity and quality of the data
  - terrestrial/airborne gravity
  - global geopotential model, digital elevation model
- inclusion of 2D and 3D crustal data
  - CRUST1, EPCrust, Global Laterally Varying Topographical Density Model TBP in 2019 (**Sheng**, **Shaw**, **Vanicek**, **Kingdon**, **Santos**, **Foroughi**)
- spatial and spectral overlaps and gaps
- quality requirements of control and validation data

Each country has to find its own financial funds.

IV. Computational

- globally, regionally and locally non-continuous surface $->$ necessity to fit
- solutions highly dependent on the selected computational method and variation
- processing large datasets
- combination of heterogeneous data types
- methodology for validation of geoid accuracy (relative and absolute)

IAG JWG 2.2.2: The 1 cm geoid experiment (Wang, Y. M.).

Results

Quasi-geoid vs geoid

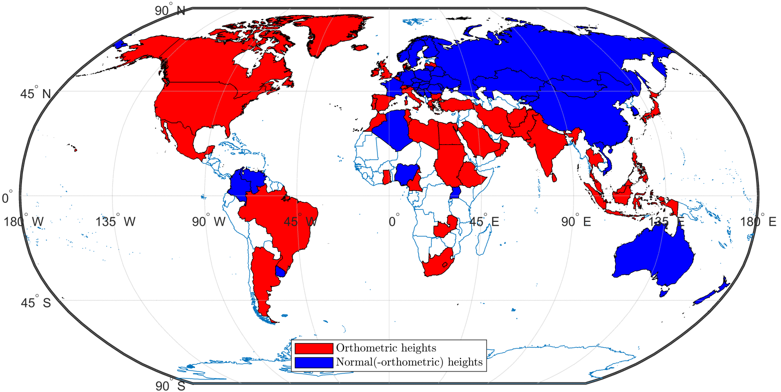


Figure 1: Orthometric or normal(-orthometric) heights for countries with published geoid or quasi-geoid model

Recent discussions: Sjöberg (2013), Vaníček et al. (2012), Foroughi et al. (2017)

The main data source: International Service for the Geoid (ISG) maintained by POLIMI, and related references

Accuracy of (quasi-)geoid models (I)

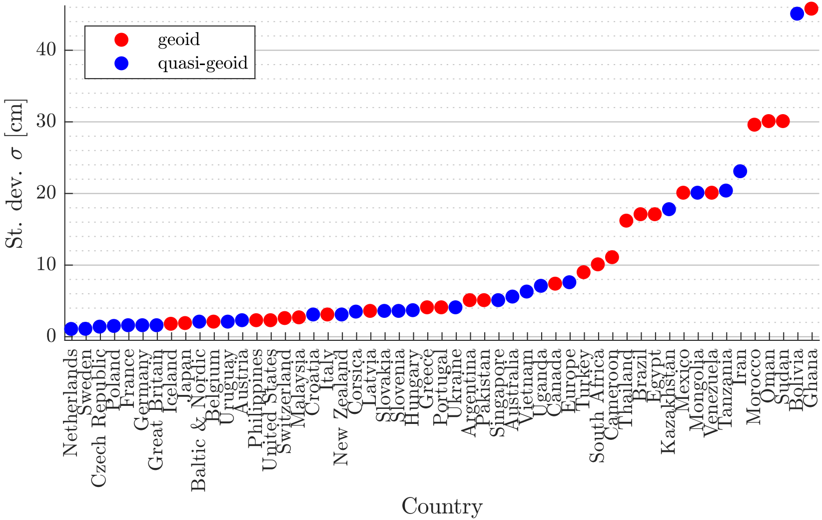


Figure 2: Accuracy of geoid and quasi-geoid models compared to GNSS/levelling for different countries.

Accuracy of (quasi)-geoid models (II)

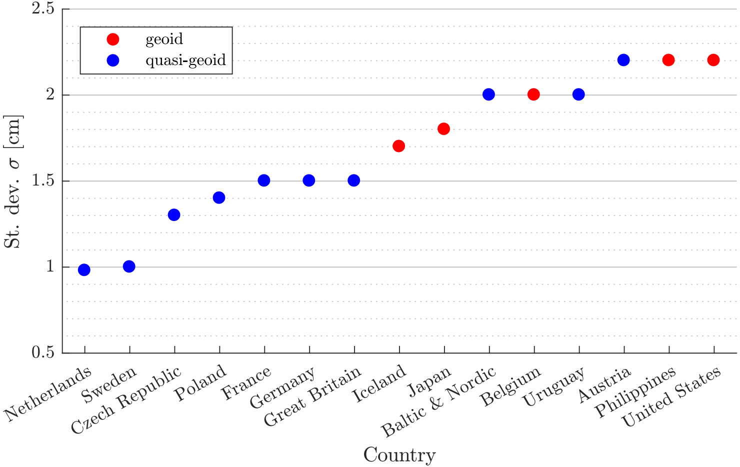


Figure 3: Accuracy of geoid and quasi-geoid models compared to GNSS/levelling for selected (most accurate) countries

Accuracy of (quasi)-geoid models (III)

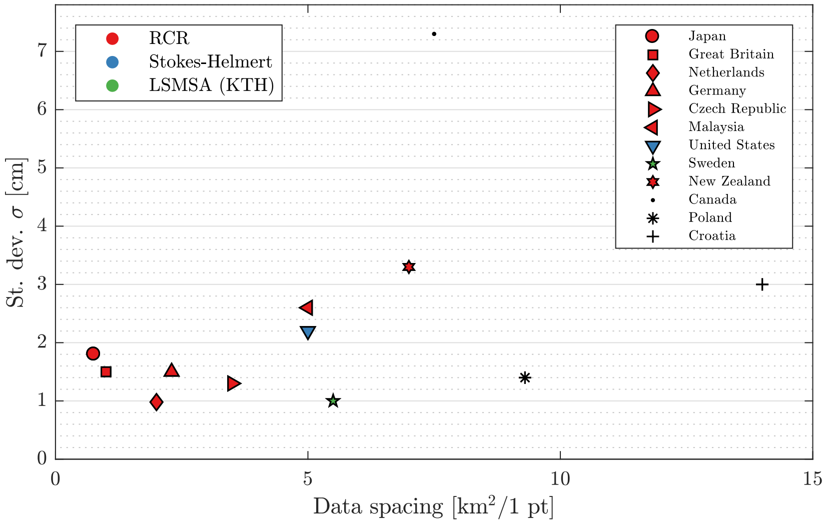


Figure 4: Data spacing vs (quasi)-geoid model accuracy. **Remark: estimated and approximate values for data spacing.**

Recent related studies: Ågren & Sjöberg (2012), Farahani et al. (2017)

Geoid computation approaches in practice

Mostly used

- Least Squares Modification of Stokes' Formula with Additive Corrections LSMSA-KTH (Sjöberg et al.)
- Remove-Compute-Restore RCR (Tscherning, Forsberg et al.)
- Stokes-Helmert (UNB S-H) (Vaníček et al.)

Computational approaches differ due...

- input data possibilities
- treatment of input data errors
- selected Stokes integral and modifications
- methods and rigorousness on the treatment of topography
- selection of integration area and modification parameters
- selection and usage of supplementary low and high-frequency data

Different approaches should produce comparable solutions (and they don't)

Comparison of geoid computational approaches (I)

Direct comparisons

identical: computation area, input data, control data

- Auvergne:
  - Quasi-geoid: RCR 3.0 cm, LSMSA 3.4 cm (1-p fit) **(Yildiz et al. 2012**
  - Quasi-geoid: RCR 3.5, LSMSA 2.9 cm (1-p fit) **(Ågren, Barzaghi, et al. 2009)**
  - Geoid: S-H 3.3 cm (no-fit) **(Janák et al. 2017)**
- Croatia: RCR 3.0 cm, LSMSA 3.5 cm **(Varga 2018)**
- Konya: RCR 9.8 cm, LSMSA 6.7 cm **(Abbak et al., 2012)**
- Sweden: RCR 32 cm, LSMSA 20 cm **(Ågren, Sjöberg, & Kiamehr 2009)**
- expected: new Colorado dataset, GEOMED-2 project

Indirect comparisons

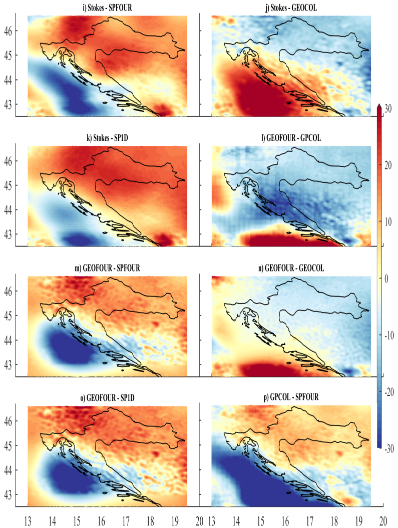
identical computation area, different input data, different control data

- Croatia: RCR 3.5 cm, LSMSA 3.9 cm **(Bašić & Bjelotomić 2014, Bjelotomić 2015)**
- Greece: RCR 4 cm, LSMSA 14 cm **(Daras 2008, Tziavos et al. 2013)**
- Poland: RCR 1.4 cm, LSMSA 2.0 cm **(Łyszkowicz 2012, Kuczynska-Siehien et al. 2016)**

Comparison of geoid computational approaches (Croatian example) (II)

Differences between RCR geoid computational methods (identical remove Δg and restore N contributions)

Geoid method	min	max	range	mean	st. dev.
Stokes - GEOFOUR	-14.8	25.3	40.1	0.9	7.5
Stokes - GPCOL	-8.0	34.2	42.2	9.6	9.2
Stokes - SPFOUR	-16.0	5.9	21.8	-3.5	4.1
Stokes - GEOCOL	-14.8	40.2	54.9	9.0	11.3
Stokes - SP1D	-15.6	4.0	19.6	-3.4	3.6
GEOFOUR - GPCOL	-7.0	34.5	41.5	8.8	8.8
GEOFOUR - SPFOUR	-39.4	16.5	55.9	-4.4	10.5
GEOFOUR - GEOCOL	-12.8	38.1	50.9	8.2	8.0
GEOFOUR - SP1D	-35.1	15.4	50.5	-4.3	9.8
GPCOL - SPFOUR	-47.3	9.3	56.6	-13.1	12.7
GPCOL - GEOCOL	-29.1	17.1	46.2	-0.6	6.3
GPCOL - SP1D	-46.1	7.0	53.1	-13.0	12.4
SPFOUR - GEOCOL	-13.9	54.1	67.9	12.5	15.0
SPFOUR - SP1D	-3.0	4.8	7.8	0.1	1.3
GEOCOL - SP1D	-52.4	13.5	65.9	-12.4	14.6



Conclusions

- scientific and civilian demands on geoid model applications and accuracy have never been higher
- each and every aspect in development of the *state-of-the-art sub-cm* geoid model needs to be reviewed and possibly improved

- as prof. A. Bjerhammar in 1969 implied: we have to be precise and specific with definitions, description and metadata about developed models in order to be able to correctly use, apply, superimpose and link them

Thank you!

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